- # Pairs of Lines
- # Angles

Power Up

facts	Power Up F
count aloud	Count by 12s from 12 to 84. Count by 5s from 3 to 53.

mental math

a. Number Sense: How many is 2 dozen? … 3 dozen? … 4 dozen?

b. Number Sense: Makalo has read 48 pages. He must read 25 more pages to finish the book. How many pages long is the book?

c. Measurement: The airplane was 1200 meters above the ground. If the airplane climbs another 340 meters, how high will it be?

d. Percent: 50% of 20

e. Percent: 25% of 20

f. Percent: 10% of 20

g. Time: How many days is 32 weeks? (*Think:* 7 × 32.)

**h. Calculation:** 4 × 9, − 1, ÷ 5, + 1, × 4

problem solving

Choose an appropriate problem-solving strategy to solve this problem. Sylvia, Goldie, and Kyle are trying to decide what to eat for lunch. Sylvia wants to eat either sandwiches or spaghetti. Goldie wants to eat salad, soup, or sandwiches. Kyle does not want to eat spaghetti but is fine with anything else. What is a lunch choice that all three can agree on?

New Concepts

Pairs of Lines

When lines cross we say that they **intersect.** If we draw two straight lines on the same flat surface, then those lines either

intersect at some point or they do not intersect at all. Lines that go in the same directions and do not intersect are called **parallel lines.** Parallel lines always stay the same distance apart. Thinking of train track rails can give us the idea of parallel lines. Here are pairs of parallel lines and parallel line segments:
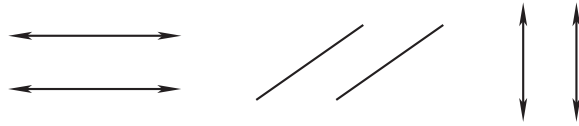
Lines on the same surface that are not parallel are called **intersecting lines.** Here are pairs of intersecting lines and intersecting line segments:
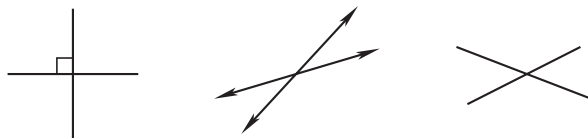
The pair of segments on the left are **perpendicular.** Perpendicular lines and segments intersect to form "square corners." The other two pairs of lines and segments are **oblique.** Oblique lines and segments are neither parallel nor perpendicular.

Activity

Parallel and Perpendicular Segments

For this activity, work with a partner. Draw a line segment. Then have your partner draw line segments, one parallel to your segment and one perpendicular to it. Switch roles with your partner and repeat the activity.

Example 1

Draw a pair of oblique lines.

We draw two lines that intersect but that do not form square corners. Many arrangements are possible.
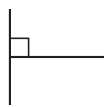
Example 2

Which of the following figures does *not* appear to contain perpendicular segments?

A   B   C   D

Perpendicular segments intersect to form square corners. The segments in **A** appear to be perpendicular. (You may need to turn your book slightly to see this.) The segments in **B** and **D** also appear to be perpendicular. The segments that do not appear to be perpendicular are those in choice **C.**

Angles

When lines or segments intersect, angles are formed. An **angle** is an "opening" between intersecting lines, rays, or segments. We see in the figures below that the amount of opening can vary; some angles may be more open or less open than other angles. We have different names for angles depending upon how open they are.
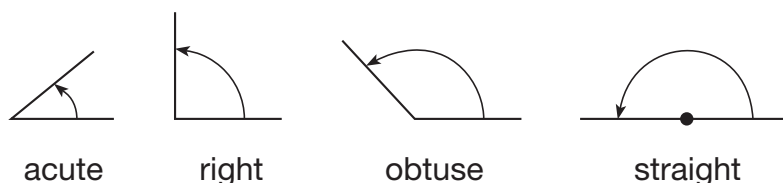
acute   right   obtuse   straight

An angle that is like the corner of a square is called a **right angle.** *Right angle* does not mean the angle opens to the right. A right angle may open in any direction. *Right angle* simply means "square corner." Sometimes we draw a small square in the angle to indicate that it is a right angle.

Right angle

Right angle

An angle whose opening is less than a right angle is an **acute angle.** Some remember this as "a cute" little angle.

An angle whose opening is more than a right angle is an **obtuse angle.**

An angle whose opening forms a straight line is a **straight angle.**

Example 3

Which of these angles appears to be a right angle?

A   B   C   D

A right angle is like the corner of a square. Angles **A** and **D** are open too wide, and angle **C** is not open wide enough. The only angle that appears to be a right angle is angle **B.**

Lesson Practice

a. Draw two parallel segments.

b. Draw two perpendicular lines.
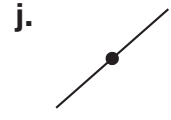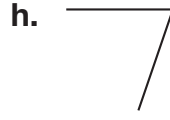
c. Draw two oblique segments.

Represent Draw an example of each angle:

d. acute angle   **e.** obtuse angle   **f.** right angle

Describe each angle shown as acute, obtuse, right, or straight:

g.   **h.**   **i.**   **j.**

Written Practice

Distributed and Integrated

1. **a.** Draw a pair of intersecting lines that are perpendicular.
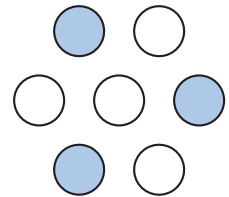(31)

     b. Draw an obtuse angle.

2. **Formulate** L'Neisha bought a kaleidoscope for $4.19. If she paid for
(16) it with a $10 bill, how much money should she get back? Translate the problem by using the *some – some went away* formula and solve the problem.

3. How many hours are there in 7 days?
(28)

4. **Formulate** From 6:00 a.m. to 4:00 p.m. the temperature rose 23° to
(11) 71°F. What was the temperature at 6:00 a.m.? Write an equation that follows the addition formula and solve the problem to find the missing number.

*5. What fraction of this group is shaded?
(30)

6. **List** Write the factors of 19.
(25)

7. $16.38
(13) $-$ \$ 9.47

8.   1000
(14) $-$ q
     576

9. $5n = 280$
(26)

10. 476
(29) $\times$ 80

11. $9.68
(29) $\times$ 60

12. $\dfrac{\$19.44}{8}$
(26)

13. Write the time that is thirty minutes before midnight.
(28)

14. Compare: $\dfrac{1}{10}$ of 100 $\bigcirc$ $\dfrac{1}{2}$ of 20
(Inv. 2)

15. A youth hockey game is divided into 3 time periods of equal length. The
(26) entire game is 36 minutes long. What is the length in minutes of each
period?

16. $96 + $128.13 + $27.49 + w = $300
(10, 13)

17. 328 $\div$ (32 $\div$ 8)
(24, 26)

18. 648 $-$ (600 + 48)
(24)

19. Think of an odd number, and multiply it by 2. Now add 1. Is the final
(2) answer odd or even?

20. Multiple Choice Which of these numbers has neither 2 nor 5 as a
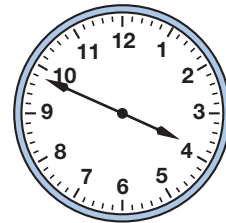(25) factor?

   A 125   **B** 251   **C** 512   **D** 215

21. It is afternoon. What time is shown on this clock?
(28)

22. What number is the numerator of the fraction $\dfrac{2}{3}$?
(Inv. 2)

*23. (**Represent**) Use words to write the number 123,400.
(7)

*24. The costs of four items in the school cafeteria are shown in this table:
(13)

Item	Cost
Cheese sandwich	$1.35
Juice	$0.60
Carrot sticks	$0.35
Apple	$0.75

Ollie purchased all four items for lunch. What was the cost of Ollie's lunch?

25. (**Estimate**) Copy this number line and draw an arrow that
(12) points to the location of the number 75.

```
   ←—+————————+————————+————————+—→
     60       70       80       90
```

26. ✎ (**Justify**) Show how to check this division answer.
(26) Is the answer correct? Why or why not?

$$\begin{array}{r} 37 \text{ R } 6 \\ 8\overline{)300} \end{array}$$

27. a. How many years is a century?
(23, 28)

b. How many years is half a century?

c. Use the numbers in the answers to parts **a** and **b** to write a fraction equal to $\frac{1}{2}$.

28. One fourth of an hour is how many minutes?
(Inv. 2, 28)

29. (**Conclude**) Write the next four terms in this counting sequence:
(1)

$$27, 18, 9, \ldots$$

30. Jackson bought five boxes of his favorite cereal for $2.87 each.
(13, 17) Altogether, how much did the five boxes of cereal cost? Change this addition problem to a multiplication problem and find the product:

$$\$2.87 + \$2.87 + \$2.87 + \$2.87 + \$2.87$$

• Polygons

Power Up

count aloud   Count by 12s from 12 to 84. Count by 5s from 4 to 54.

mental math

a. **Time:** How many hours are in 2 days?

b. **Number Sense:** There are 39 boys and 45 girls on the playground. Altogether, how many children are on the playground?

c. **Money:** Jean earned $680. She spent $400. How much money does Jean have left?

d. **Time:** 60 minutes is 1 hour. How many minutes is 10 hours?

e. **Percent:** 50% of 30

f. **Percent:** 10% of 30

g. **Number Sense:** 8×45

h. **Calculation:** $6 \times 7, -2, \div 5, \times 2, -1, \div 5$

problem solving

Choose an appropriate problem-solving strategy to solve this problem. At the store a pencil costs 34¢, an eraser costs 52¢, and a notebook costs $1.05. Santo purchased a combination of five of these items for a total of $2.59. What did Santo purchase?

New Concept

A **plane** is a flat surface that extends without end. The classroom floor is part of a plane that extends beyond the walls. The wall surfaces in the room are parts of other planes. Planes can contain flat shapes such as triangles, squares, and circles. Some of these flat shapes are **polygons.**

A polygon is a flat shape formed by line segments that close in an area. Each of these shapes is a polygon:

The line segments that form a polygon are called **sides.** Two sides meet at a **vertex** (plural: *vertices*) to form an angle. A polygon may have three or more straight sides, and it has as many vertices and angles as it has sides.

Polygons do not have any curved sides.

Conclude These figures are not polygons. Explain why.

Connect Name a real-world example of a polygon. Explain your reasoning.

Polygons are named by the number of sides they have. The table below names some common polygons.

Polygons

Polygon	Number of Sides	Example
Triangle	3	
Quadrilateral	4	
Pentagon	5	
Hexagon	6	
Heptagon	7	
Octagon	8	
Decagon	10	
Dodecagon	12	

Notice that a four-sided polygon is a **quadrilateral.** There are different kinds of quadrilaterals, such as squares, rectangles, parallelograms, and trapezoids. We will study these classifications in more detail later.

Example 1

This figure is an example of a quadrilateral:
Which of these shapes is *not* a quadrilateral?

A   B   C   D
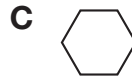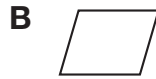
A quadrilateral is a polygon with four sides. The shape that does not have four sides is choice **C.**

(**Conclude**) Which of the shapes in choices **A–D** appear to have perpendicular sides?

Sometimes we enclose an area by using smooth curves. A circle is one example of an area that is enclosed by a smooth curve. Because a circle does not enclose an area with line segments, a circle is not a polygon.

Example 2

Which of these shapes is not a polygon?

A   B   C   D
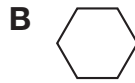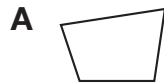
A polygon is formed by line segments. A circle is a smooth curve. The shape that is not a polygon is choice **C.**

(**Conclude**) Which of the shapes in choices **A–D** appear to have at least two parallel sides?

Example 3

Name each of these polygons:

a.   b.   c.

    a. The polygon has six sides. It is a **hexagon.**

    b. This 12-sided polygon is a **dodecagon.**

    c. The block-letter T has 8 sides and is an **octagon.**

Thinking Skill

(**Connect**)

Name a shape that has all sides congruent.

Figures that have the same size and shape are **congruent.** The three triangles below are congruent even though they have been flipped and turned to different positions.
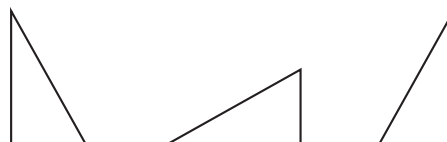
Example 4

Which two rectangles below are congruent?

A ▭ B ▭ C ▯ D ▭

Rectangles **A** and **C** have the same size and shape, so they are congruent figures.

Thinking Skill

Connect

Suppose two figures are identical. Are the figures congruent, similar, or both?

Congruent figures are also **similar.** Similar figures have the same shape. They may or may not be the same size. When looking at two similar figures that are not the same size, the larger figure will look like a magnified version of the smaller figure. These two triangles are similar but not congruent.
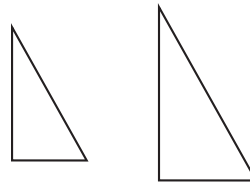
Example 5

Which two triangles below are similar?

A ◺ B △ C ◹ D △

Triangles **B** and **D** have the same shape, so they are similar figures.

Lesson Practice

a. Draw a triangle with two perpendicular sides.

b. A quadrilateral is a polygon with how many sides?

c. Draw a quadrilateral that has one pair of parallel sides.

d. Draw a quadrilateral with two pairs of parallel sides.

e. Draw a quadrilateral that has no parallel sides. (Begin by drawing two nonparallel segments. Then connect those with two other nonparallel segments.)

Name each shape:

f. **g.** **h.**
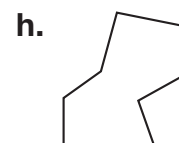
i. (Represent) Draw a polygon shaped like the block letter F. What type of polygon did you draw?

j. (Represent) Draw two triangles that are congruent.

Written Practice   *Distributed and Integrated*

1. (Analyze) Suki took $20 to the carnival. She spent $\frac{1}{2}$ of her money on
(Inv. 2) rides, $\frac{1}{4}$ of her money on food, and $\frac{1}{10}$ of her money on parking. How much did Suki spend on rides? … on food? … on parking?

(Formulate) For problems **2–4,** write an equation and find the answer.

2. Hank says that the horse trough holds 18 buckets of water. If a bucket
(21) holds 3 gallons, how many gallons does the trough hold?

3. Kareem chopped a tree that was 52 feet tall into four logs of equal
(21) length. How many feet long was each log?

*4. After 20 minutes Carlotta had answered 17 of the 45 questions on the
(16) test. How many questions remained for Carlotta to answer?

5. How many seconds are in 1 hour?
(28, 29)

6. $56.37   **7.**   5286   **8.** $40.00   **9.**   67
(13) $34.28   *(14)* $- \quad k$ *(13)* $- 39.56 *(10)*   72
 $+ $ 9.75$ 4319   43
                                                                 91
                                                                 48
10. $936 \div (36 \div 9)$ **11.**   596   19
(24, 26)   *(29)* $\times \quad 600$ 648
                                                                $+ \quad m$
12. $\frac{$46.56}{8}$ **13.** $4.07 \times 80   996
(26)   *(29)*

14. $9 \times 12 \times 0$ **15.** $936 \div 7$
(15, 18)   *(26)*

16. Compare: $\frac{1}{3}$ of 60 $\bigcirc$ $\frac{1}{5}$ of 100
(Inv. 3)

*17. Multiple Choice Which of these angles does not appear to be a right angle?
(31)

A   B   C   D
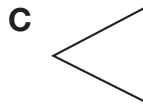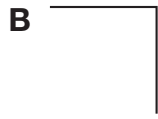
18. ✏️ **List** Which factors of 18 are also factors of 36? Explain how you
(25) know.

19. What fraction of the rectangle is shaded? What percent
(30) of the rectangle is shaded?
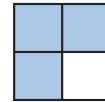
*20. Represent Draw a horizontal line segment and a vertical line segment
(31) that intersect.

21. According to this calendar, July 17, 2025, is what
(28) day of the week?

JULY 2025					
M	T	W	T	F	
	1	2	3	4	5
6	7	8	9	10	11	12
13	14	15	16	17	18	19
20	21	22	23	24	25	26
27	28	29	30	31		

22. What is the name for the bottom number of a fraction?
(Inv. 2)

23. Connect Write two multiplication facts and two division facts for the
(19) fact family 9, 10, and 90.

*24. Conclude What are the next three terms in this counting sequence?
(1)

..., 660, 670, 680, ____, ____, ____, ...

25. From morning until evening of one day, the temperature increased
(27) 17°F. The evening temperature was 54°F. What was the morning
temperature?

26. Represent Draw a polygon shaped like a block letter H. What type of
(32) polygon did you draw?

27. Show how to check this division answer. Is the answer correct?
(26)

$$\begin{array}{r} 57 \text{ R } 1 \\ 7\overline{)400} \end{array}$$

*28. (**Analyze**) Use the digits 0, 2, and 5 to make a three-digit number that
(25) has both 2 and 5 as factors.

29. a. How many is a dozen?
(2, 23)

b. How many is half a dozen?

c. Use the answers to parts **a** and **b** to write a fraction equal to $\frac{1}{2}$.

30. Halley's Comet was last seen in 1986. The next opportunity to see it
(11) will be 76 years later. What year presents the next opportunity to see
Halley's Comet?

Early Finishers
Real-World Connection

Find four classroom objects that are examples of different types of polygons. (*Hint:* Remember that a polygon is flat.)

a. Draw a picture of each object. Be sure to show the correct number of sides.

b. Label each drawing with its polygon name.

c. Choose two of the polygons that you found. Explain how they are alike and how they are different.

• Rounding Numbers and Estimating

Power Up

facts Power Up F

count aloud Count by 12s from 12 to 96. Count by 6s from 6 to 96.

mental math

a. **Time:** How many months are in half a year? … a year and a half?

b. **Money:** How much money is 4 quarters? … 5 quarters? … 6 quarters?

c. **Money:** The cost of one video game is $42. What is the cost of ten video games?

d. **Percent:** 50% of 40

e. **Percent:** 25% of 40

f. **Percent:** 10% of 40

g. **Money:** The regular price is $8.40. If the item is on sale for $1.40 off, what is the sale price?

h. **Calculation:** $8 \times 8, - 1, \div 9, \times 3, - 1, \div 4$

problem solving

Choose an appropriate problem-solving strategy to solve this problem. Julio identified a pattern in the lunches served at the school cafeteria. He found that macaroni was served as part of every eighth lunch. If macaroni was last served on a Friday, what are the next two days of the week Julio can expect macaroni to be served? (School is in session Monday through Friday with no holidays.)

When we do not need an exact amount to describe a situation, we can round numbers to a specific place value.

The attendance at the game was 614.   614 was rounded to
About 600 people attended the game.   the nearest hundred.

The price of the shoes was $48.97. $48.97 was rounded to
The shoes cost about $50.   the nearest ten dollars.

Numbers that have been rounded usually end in one or more zeros. When we want to estimate, sometimes we use numbers that have been rounded to a specific place.

When we *round* a number, we find another number to which the number is near. One way we can do this is with a number line. To round 67 to the nearest ten, for example, we find the multiple of ten that is nearest to 67. On the number line below we see that 67 falls between 60 and 70.

$$59 \quad \circled{60} \quad 61 \quad 62 \quad 63 \quad 64 \quad 65 \quad 66 \quad \boxed{67} \quad 68 \quad 69 \quad \circled{70} \quad 71$$
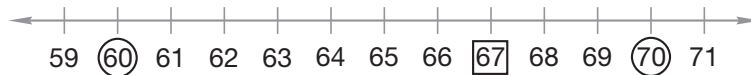
Since 67 is nearer to 70 than to 60, we *round up* to 70.

When the number we are rounding is halfway between two multiples of ten, we usually round up to the larger number. Sixty-five is halfway between 60 and 70, so we would round 65 to 70. Likewise, 450 is halfway between 400 and 500, so we would round 450 to 500.

Example 1

Round 523 to the nearest hundred.

When we round a number to the nearest hundred, we find the multiple of 100 to which it is nearest. Recall that the multiples of 100 are the numbers we say when we count by hundreds: 100, 200, 300, 400, and so on. We use a number line marked and labeled with hundreds to picture this problem.

$$523$$
$$\downarrow$$
$$400 \qquad 500 \qquad 600$$
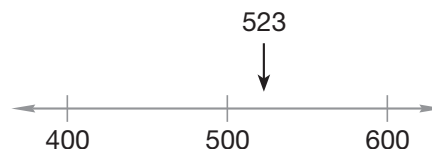
Placing 523 on the number line, we see that it falls between the multiples 500 and 600. Since 523 is nearer to 500 than to 600, we *round down* to **500.**

Rounding can help us **estimate** the answer to a problem. One way we estimate is by calculating with **round numbers** that are easy to use.

Example 2

In a presidential election, each state is assigned a certain number of electoral votes. Based on the 2000 Census, the state of Florida was assigned 27 electoral votes and the state of Texas was assigned 34 electoral votes. About how many electoral votes can a candidate win altogether in those states?

The word "about" means we may estimate. We will round each number to the nearest ten. Since 27 rounds to 30 and 34 rounds to 30, we estimate that a candidate can win **about 60 electoral votes** in Florida and Texas.

Example 3

The average depth of the East China Sea is 620 feet. The average depth of the Yellow Sea is 121 feet. To the nearest hundred feet, about how many feet deeper is the average depth of the East China Sea?

We round both numbers to the nearest hundred and then find the difference.

620 rounds to 600.

121 rounds to 100.

$600 - 100 = 500$

The average depth of the East China Sea is about **500 feet** deeper than the average depth of the Yellow Sea.

Example 4

A community theater group is giving four performances of a play. The theater can seat 424 guests. All of the tickets for each performance have been sold. Estimate the total number of guests who plan to see the performances by first rounding the number of guests for each performance to the nearest hundred.

We round 424 to 400 before multiplying.

$400 \times 4 = 1600$

About **1600 guests** plan to see the performance. The exact answer is 1696, so we see that our estimate is close to the exact answer.

When estimating, it is not necessary to use numbers rounded to the nearest ten or hundred or thousand. Instead we may use **compatible numbers,** which are numbers that are easy to compute mentally.

Consider Example 4 again. In that problem, 424 is multiplied by 4. Since 25 is easy to mentally multiply by 4 (think of 4 quarters), we could replace 424 with 425 and multiply by 4 to estimate the product.

$$4 \times 400 = 1600$$
$$4 \times 25 \ \ = 100$$
$$\text{so } 4 \times 425 = 1700$$

In this case, using the compatible number 425 results in an estimate that is close to the exact answer.

Example 5

Lela's car traveled 129 miles and used 4 gallons of gas. About how many miles did the car travel on each gallon?

We will use compatible numbers to estimate. Instead of changing 129 to 130, we change 129 to 128 and divide by 4.

$$128 \div 4 = 32$$

Lela's car traveled about **32 miles per gallon.**

Lesson Practice

Represent Round to the nearest ten. For each problem, you may draw a number line to help you round.

a. 72   **b.** 87   **c.** 49   **d.** 95

Represent Round to the nearest hundred. For each problem, you may draw a number line to help you round.

e. 685   **f.** 420   **g.** 776   **h.** 350

i. **Estimate** A row of bleachers in a gymnasium can seat 96 people. Estimate the total number of people who can be seated in eight rows. Explain your reasoning.

j. **Estimate** In their Major League Baseball careers, Lou Gehrig hit 493 home runs and Frank Robinson hit 586 home runs. About how many home runs did those two players hit altogether?

1. **Represent** Draw a pair of vertical parallel lines.
(31)

2. **a.** Round 537 to the nearest hundred.
(33)

  b. Round 78 to the nearest ten.

3. Use your answer to problem **2** to estimate the product of 537
(33) and 78.

4. **Analyze** Forty animals were brought to the pet show. One half were
(Inv. 2) mammals, one fourth were fish, one tenth were reptiles, and the rest
  were birds. How many mammals were brought to the pet show? How
  many fish? How many reptiles? How many birds?

5. Keisha was standing in a line that had 20 people in it (including herself).
(7) If 5 people were in front of her, how many people were behind her?

Formulate For problems **6–8,** write an equation and find the answer.

6. Seven hours is how many minutes?
(28)

7. After paying $7.50 for a movie ticket, Salvador still had $3.75. How
(16) much money did Salvador have before paying for the ticket?

*8. **Estimate** The Price family set the odometer to zero before they started
(11) their trip. By the end of the first day, the Price family had traveled
  427 miles. By the end of the second day, they had traveled a total of
  902 miles. About how far did the Price family travel the second day?

9. $34.28
(13) $ 9.76
  + $20.84

10. 3526
(14) $- \quad v$
  1617

11. $10.00
(13) $- $ 0.86$

12. 499
(6) 25
  43
  756
  67
  94
  + 32

13. 563
(29) $\times \quad 90$

14. $2.86
(29) $\times \quad 70$

15. 479
(29) $\times \quad 800$

16. $3\overline{)1122}$
(26)

17. $6m = \$5.76$
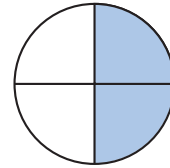(26)

18. $10\overline{)2735}$
(26)

19. $64.23 + $5.96 + $17 + ($1 − $0.16)
(13, 24)

20. From March 1 to December 1 is how many months?
(28)

*21. a. What fraction of the circle is shaded?
(30, Inv. 3)

    b. What decimal part of the circle is shaded?

    c. Is more than or less than 25% of the circle shaded?

22. Multiple Choice Which word means "parallel to the horizon"?
(31)

    A vertical   **B** oblique   **C** horizontal   **D** perpendicular

23. Write the time that is a quarter after one in the afternoon.
(28)

24. (**Represent**) Draw a horizontal number line from 0 to 50 with only zero
(12) and tens marked and labeled.

25. (**Predict**) What is the tenth term of this counting sequence?
(1)

$$7, 14, 21, \ldots$$

26. (**Represent**) Draw an acute angle.
(31)

*27. (**List**) Write the factors of both 7 and 28. Circle the common factors.
(25)

*28. Multiple Choice At which of these times are the hands of a clock
(28, 31) perpendicular?

    A 6:00   **B** 12:30   **C** 9:00   **D** 2:00

29. Main Street and Allen Street intersect at a traffic light. The two streets
(31) form square corners where they meet.

    a. Draw segments to show how Main Street and Allen Street meet.

    b. Multiple Choice Which of these words best describes the
        segments in your drawing?

        A parallel   **B** perpendicular   **C** oblique   **D** horizontal

*30. (**Estimate**) In 1804 Lewis and Clark, along with their guide, Sacagawea, explored the
(33) northwestern United States. About how many years ago did that exploration take
place?

• Division with Zeros in the Quotient

Power Up

count aloud   Count by 12s from 12 to 108.

mental math
a. **Time:** How many years is half a century? . . . half a decade?

b. **Estimation:** Round 48 to the nearest ten.

c. **Money:** One roll of pennies is 50¢. Eight rolls of pennies is how much money?

d. **Money:** Four quarters is $1.00. How many quarters equal $25?

e. **Percent:** 50% of $50

f. **Percent:** 10% of $50

g. **Measurement:** Casius walked 1420 feet to get to the park. Then he walked another 300 feet to reach the pond. How far did Casius walk?

h. **Calculation:** $3 \times 8, + 1, \times 2, - 1, \div 7, \div 7$

problem solving   Choose an appropriate problem-solving strategy to solve this problem. If a coin is flipped twice, it may land heads then heads, heads then tails, tails then heads, or tails then tails. If a coin is flipped three times, what are the possible orders in which it could land?

New Concept

Recall that the answer to a division problem is called a quotient. Sometimes when we divide, one or more of the digits in the quotient is a zero. When this happens, we continue to follow the four steps in the division algorithm: divide, multiply, subtract, and bring down.

Divide: $6\overline{)365}$

We begin by breaking the division problem into a smaller problem: $6\overline{)36}$.

$$6\overline{)365}$$

Then we divide, multiply, subtract, and bring down. When we subtract, we get zero, which we may or may not write, and we bring down the 5. Since there is a number to bring down, we divide again. The new division is $6\overline{)5}$.

$$\begin{array}{r} 6 \\ 6\overline{)365} \\ -36\downarrow \\ \hline 05 \end{array}$$

Since we cannot divide 5 by 6 even once, we write a zero in the quotient, multiply, and subtract. Since there is no other number to bring down, the division is finished and the remainder is 5. Our answer is **60 R 5.**

$$\begin{array}{r} 60 \text{ R } 5 \\ 6\overline{)365} \\ -36 \\ \hline 5 \\ -0 \\ \hline 5 \end{array}$$

Explain how to determine if the answer is reasonable.

Again, we check a division answer by multiplying the quotient by the divisor and then adding the remainder to this product.

$$\begin{array}{r} 60 \\ \times 6 \\ \hline 360 \\ +5 \\ \hline 365 \end{array}$$

Since the result, 365, equals the dividend, we can be confident that our answer is correct.

A train traveled 630 kilometers in 6 hours. To find the average speed of the train, divide 630 km by 6 hours.

We break the division problem into smaller problems. We can find $6\overline{)6}$, so we divide, multiply, subtract, and bring down. The next division is $6\overline{)3}$.

$$\begin{array}{r} 1 \\ 6\overline{)630} \\ -6\downarrow \\ \hline 03 \end{array}$$

Since the dividend (number we are dividing) is less than the divisor (number we are dividing by), we write a zero in the quotient. Then we multiply, subtract, and bring down. The next division is $6\overline{)30}$.

$$\begin{array}{r} 10 \\ 6\overline{)630} \\ -6 \\ \hline 3 \\ -0 \\ \hline 30 \end{array}$$

We divide 30 by 6, multiply, and subtract. Since there is no other number to bring down, the division is finished. When we divide 630 into 6 equal parts, the quotient is 105. This means the train averaged **105 kilometers per hour.**

$$\begin{array}{r} 105 \\ 6\overline{)630} \\ \underline{-\ 6} \\ 3 \\ \underline{-\ 0} \\ 30 \\ \underline{-\ 30} \\ 0 \end{array}$$

Example 3

An object on the moon has a different weight than that same object on Earth. To estimate the weight of an object on the moon, divide its weight on Earth by 6.

Jason weighs 115 pounds on Earth. Estimate the number of pounds he would weigh on the moon.

We round 115 pounds to 120 pounds, which is easy to divide by 6. We find that Jason would weigh **about 20 pounds** on the moon.

Lesson Practice   Divide:

a. $3\overline{)61}$ **b.** $6\overline{)242}$ **c.** $3\overline{)121}$ **d.** $4\overline{)1628}$

e. $4\overline{)122}$ **f.** $5\overline{)\$5.25}$ **g.** $2\overline{)\$6.18}$ **h.** $6\overline{)4981}$

i. $10\overline{)301}$ **j.** $4\overline{)\$8.24}$ **k.** $7\overline{)\$5.60}$ **l.** $8\overline{)4818}$

m. Show how to check this division answer. Is the answer correct?

$$\begin{array}{r} 108 \text{ R } 2 \\ 6\overline{)650} \end{array}$$

n. **Estimate** Therin needs to refill the 7 printers in the school office. She has 500 sheets of paper. She wants to put about the same number of sheets into each printer. Estimate about how many pages Therin will put into each printer. Explain why your answer is reasonable.

Written Practice   *Distributed and Integrated*

*1. **Represent** Draw a horizontal line. Draw another line that is perpendicular to the horizontal line.
(31)

*2. **Analyze** One hundred students named their favorite vegetable. One
(Inv. 2)   half named "carrots," one fourth named "broccoli," one tenth named "peas," and the rest named "celery." How many students named carrots? … broccoli? … peas? … celery?

3. What year was one century after the California gold rush began
(28) in 1849?

(**Formulate**) For problems **4–6,** write an equation and find the answer.

4. How many minutes are in one day?
(28, 29)

5. A hen laid 10 dozen eggs in one year. How many eggs is that?
(21, 29)

6. A group of friends is canoeing 300 miles of Mississippi's Tombigbee
(16) River. The group has already canoed 127 miles. How many miles must
the group still travel to reach their destination?

7. $6\overline{)365}$
(34)

8. $6\overline{)\$6.36}$
(34)

9. $5\overline{)536}$
(34)

10. $10\overline{)653}$
(26)

11. $4\overline{)\$4.36}$
(34)

12. 95×500
(29)

* **13.** Round 83 to the nearest ten.
(33)

14. $345 + 57 + 760 + 398 + 762 + 584 + w = 3000$
(10)

15. $3004 - (3000 - 4)$
(24)

16. $\$5.93 \times 40$
(29)

17. Compare: $\frac{1}{3}$ of 12 $\bigcirc$ $\frac{1}{8}$ of 24
(Inv. 3)

18. $\$12 + \$8.75 + \$0.96$
(13)
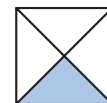
19. $\$20 - \$12.46$
(13)

20. $8 \times 30 \times 15$
(18, 29)

21. $6 \times 7 \times 8 \times 9$
(18)

22. (**Conclude**) What are the next three terms in this counting sequence?
(1)

$$\ldots, 460, 470, 480, \underline{\quad}, \underline{\quad}, \underline{\quad}, \ldots$$

23. What fraction of this square is shaded? What decimal
(30) part of the square is shaded?

* **24.** **Multiple Choice** If two segments that intersect are perpendicular,
(31) then what kind of angle do they form?

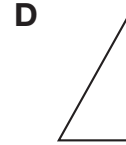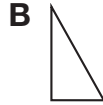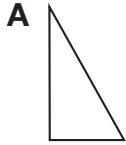**A** acute **B** right **C** obtuse **D** straight

25. A team practice was scheduled for 10:00 a.m. This clock
(28) shows the time that Suzanne arrived at practice. What
time did Suzanne arrive?

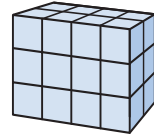***26. Multiple Choice** Which two triangles appear to be congruent?
(32)

A   B   C   D

27. Show how to check this division answer. Is the answer
(26) correct?

$$\begin{array}{r} 84 \text{ R } 8 \\ 9\overline{)764} \end{array}$$

*28. ✏️ **Explain** K'Mara calculated the number of blocks in this
(24) figure by finding $(2 \times 3) \times 4$. Mateo found the number of
blocks by finding $2 \times (3 \times 4)$. Who was correct? Explain why.

29. Multiple Choice Without dividing, decide which of these division
(22) problems will *not* have a remainder.

A $\dfrac{49}{2}$ B $\dfrac{52}{5}$ C $\dfrac{600}{10}$ D $\dfrac{71}{3}$

30. ✏️ **Estimate** An employee works 5 days each week and for 7 hours
(33) and 45 minutes each day. Estimate the number of hours the employee
works each week. Explain your estimate.

• Word Problems About Comparing and Elapsed Time

Power Up

facts	Power Up F
count aloud	Count by 6s from 6 to 96.
mental math	**a. Estimation:** Round 285 to the nearest hundred.
	b. Number Sense: $300 + 800$
	c. Number Sense: 300×8
	d. Number Sense: There are 42 beads in each necklace. How many beads are in 5 necklaces? (*Think:* 5×42 or half of 10×42.)
	e. Fractional Parts: $\frac{1}{2}$ of 42 meters
	f. Percent: 50% of $8.00
	g. Percent: 25% of $8.00
	h. Calculation: $3 \times 9, + 1, \div 7, + 1, \times 5, - 1, \div 4$
problem solving	Choose an appropriate problem-solving strategy to solve this problem. Montrelyn has a pet dog and a pet cat. Together the two animals weigh 42 pounds. The dog weighs 14 pounds more than the cat. How much does each pet weigh?

New Concept

Numbers are used to describe the quantity of objects.

> *Seven different containers are on a loading dock awaiting shipment.*

Numbers are also used to describe the size of objects.

> *The heaviest container weighs 245 pounds.*

Some word problems compare numbers of objects or sizes of objects.

The containers range in weight from 245 pounds to 160 pounds. The heaviest container weighs how many pounds more than the lightest container?

In comparison problems, one number is larger and another number is smaller. Drawing a sketch can help us understand a comparison story. We will draw two rectangles, one taller than the other. Then we will draw an arrow from the top of the shorter rectangle to extend as high as the taller rectangle. The length of the arrow shows the difference in height between the two rectangles. The two rectangles and the arrow each have a circle for a number. For this problem, the rectangles stand for the weights of the two containers.

Heaviest container   Lightest container
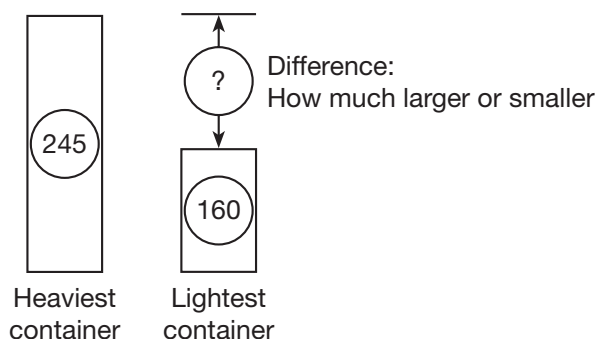
A comparison problem may be solved by using a subtraction formula. If we subtract the smaller number from the larger number, we find the difference between the two numbers. Here we show two ways to write a comparison equation:

$$\begin{array}{r} \text{Larger} \\ - \ \text{Smaller} \\ \hline \text{Difference} \end{array} \qquad \text{Larger} - \text{Smaller} = \text{Difference}$$

In this problem the number missing is the difference, which we find by subtracting.

$$\begin{array}{r} \overset{1}{2}\,\overset{1}{4}\,5 \text{ pounds} \\ - \ 1\ 6\ 0 \text{ pounds} \\ \hline 8\ 5 \text{ pounds} \end{array}$$

We find that the heaviest container weighs 85 pounds more than the lightest container.

Justify Explain how to decide if the answer is reasonable.

Example 1

Mel is 6 years younger than his brother Cleon. Mel is 11 years old. How old is Cleon?

We will draw two rectangles to illustrate the problem. The rectangles stand for the boys' ages. Since Mel is younger, his rectangle is shorter.

Difference:
How much older or ounger

Cleon   Mel
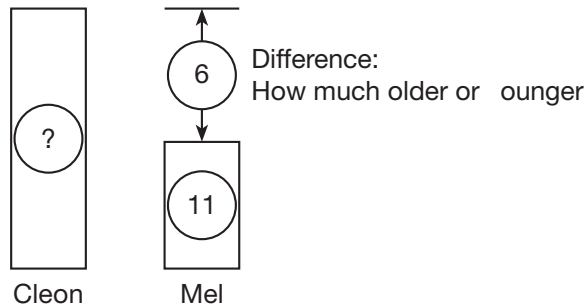
From the story we know that Mel is 11 and that the difference between his age and his brother's age is 6 years. We write the numbers in the circles and use a subtraction pattern to solve the problem.

$$\text{Larger} - \text{Smaller} = \text{Difference}$$

$$g - 11 = 6$$

We find the first number of a subtraction problem by adding

$$\begin{array}{r} g \\ -\ 11 \\ \hline 6 \end{array} \quad \text{so} \quad \begin{array}{r} 6 \\ +\ 11 \\ \hline g \end{array}$$

Adding 6 and 11, we find that Cleon is **17 years old.**

Verify Why can we use addition to solve a subtraction problem?

Elapsed-time problems are like comparison problems. **Elapsed time** is the amount of time between two points in time.

Elapsed time

Earlier   Later
time   time
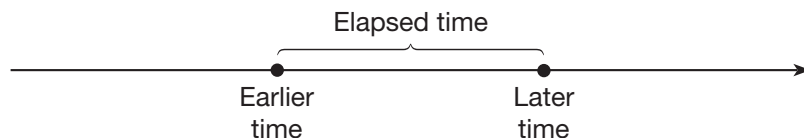
Your age is an example of elapsed time. Your age is the difference between the present time and the time of your birth. To calculate elapsed time, we subtract the earlier time from the later time. Below are two forms of the equation. We use the word *difference* for elapsed time.

$$\begin{array}{r} \text{Later} \\ -\ \text{Earlier} \\ \hline \text{Difference} \end{array} \qquad \text{Later} - \text{Earlier} = \text{Difference}$$

Example 2

How many years were there from 1492 to 1620?[1]

To find the number of years from one date to another, we may subtract. We subtract the earlier date from the later date. In this problem we subtract 1492 from 1620 and find that there were **128 years** from 1492 to 1620.

$$
\begin{array}{r}
5\ 1\\
1\,\cancel{6}\,\cancel{2}\,^1\!0\\
-\ 1\ 4\ 9\ 2\\
\hline
1\ 2\ 8
\end{array}
$$

Example 3

A school day at Edison School begins at 8:10 a.m. The school day ends at 3:15 p.m. What is the length of a school day at Edison School?

We can solve elapsed time problems using the *later − earlier = difference* equation. The later time is 3:15 p.m., which can also be written as 15:15. The earlier time is 8:10.

$$
\begin{array}{lr}
\text{Later} & 15{:}15\\
-\ \text{Earlier} & -\ \ 8{:}10\\
\hline
\text{Difference} & 7{:}05
\end{array}
$$

We find that the length of the school day is **7 hours 5 minutes.**

Lesson Practice

Formulate For problems **a–e,** write an equation and solve the problem.

a. There were 4 more boys than girls in the class. If there were 17 boys in the class, how many girls were there?

b. The Mackinac Bridge spans 3800 feet, which is 400 feet less than the span of the Golden Gate Bridge. What is the span of the Golden Gate Bridge?

c. From Rome to Paris is 1120 kilometers. From Rome to London is 1448 kilometers. The distance from Rome to London is how much greater than the distance from Rome to Paris?

d. How many years were there between the Magna Carta in 1215 and the Declaration of Independence in 1776?

e. Elena finished her homework at 8:05 p.m. She began at 6:50 p.m. How long did it take Elena to complete her homework?

---

[1] Unless otherwise specified, all dates in this book are A.D.

*1. **Represent** Draw a pair of intersecting oblique lines.
(12, 31)

Formulate For problems **2–5,** write an equation and find the answer.

2. **Justify** In three games Tamyra's bowling scores were 109, 98,
(11, 33) and 135. What was her total score for all three games? Explain why
your answer is reasonable.

3. Santiago is 8 inches taller than Sancha. If Santiago is 63 inches
(35) tall, how tall is Sancha?

4. How many years were there from the time the Statue of Liberty was
(35) built in 1886 to the centennial ceremony in 1986?

5. The toll for one car to cross the bridge was $1.50. In ten minutes,
(21, 29) 40 cars crossed the bridge. What was the total toll for the 40 cars?

6. What is the product of nine hundred nineteen and ninety?
(29)

*7. a. Multiple Choice Which two quadrilaterals appear to be similar?
(31, 32)

A **B** **C** **D**
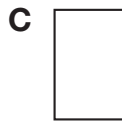
  b. Which shape above does not appear to have any perpendicular
sides?

8. **List** Write the factors of 18 and 28. Circle the common factors.
(25)

9. $4m = 432$ **10.** $423 \div 6$
(26, 34)   (34)

11. $243 \div 8$ **12.** $2001 \div 4$
(34)   (34)

13. $1020 \div 10$ **14.** $420 \div (42 \div 6)$
(34)   (24, 34)

15. Round 468 to the nearest hundred.
(33)

16. 4657
(6)　285
　　+ 1223

17. 3165
(9)　− 1635

18. $10.00
(13)　− $ 8.93

19. 436
(29)　× 70

20. $8.57
(17)　× 7

21. 600
(29)　× 900

*22. Write the shaded part of this rectangle as a fraction, a
(30, Inv. 3) decimal, and a percent. Is more than or less than 50% of the rectangle shaded?
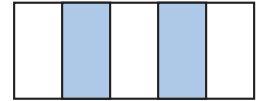
23. This morning the school bus picked Tevin up at 6:55 a.m. and
(28) arrived at Tevin's school at 7:48 a.m. How much time did Tevin spend on the bus this morning?

24. From November 1 of one year to March 1 of the next year is how many
(28) months?

25. (**Conclude**) What are the next three terms in this counting sequence?
(1)

..., 1900, 2000, 2100, ____, ____, ____, ...

26. Show how to check this division answer. Is the answer
(26) correct?

$$6\overline{)432}$$ → 72

27. One half of the 14 animals that were boarded at a kennel last
(2) weekend were dogs. How many dogs were boarded at the kennel last weekend?

28. Use words to name the number 68,200.
(7)

29. a. (**Represent**) Draw a right angle.
(31)

b. Draw an acute angle whose opening is half the size of a right angle.

30. (**Estimate**) A tennis team began the season with about 175 new
(33, 34) tennis balls. The balls were purchased in containers called sleeves, with 3 balls in each sleeve. About how many sleeves of balls did the team purchase? Explain why your estimate is reasonable.

• Classifying Triangles

Power Up

facts	Power Up E
count aloud	Count up by 7s from 7 to 84.
mental math	

   a. **Estimation:** Round 73 to the nearest ten.

   b. **Number Sense:** $70 + 80$

   c. **Number Sense:** 70×8

   d. **Number Sense:** A spider has 8 legs. How many legs do 73 spiders have? (*Think:* 8×73.)

   e. **Fractional Parts:** One day is 24 hours. How many hours is $\frac{1}{2}$ of a day?

   f. **Percent:** 50% of $12

   g. **Percent:** 25% of $12

   h. **Time:** It took Derek 360 seconds to climb the hill. It took him 200 seconds to get back down. Altogether, how many seconds did it take Derek to climb the hill and then get back down?

   i. **Calculation:** $9 \times 6, + 2, \div 7, + 1, \times 4, \div 6$

problem solving	Choose an appropriate problem-solving strategy to solve this problem. There are three crosswalk signals between Simone's home and school. When Simone comes to a signal, she either walks through the crosswalk or waits for the signal to turn. List the eight possible patterns of signals for Simone's walk to school. Use the words "walk" and "wait."

New Concept

In this lesson we will learn the names of different kinds of triangles.

All **triangles** have three angles and three sides, but we can *classify,* or sort, triangles by the size of their angles and by the relative lengths of their sides.

Recall from Lesson 31 that three types of angles are acute angles, right angles, and obtuse angles.

| acute angle | right angle | obtuse angle |
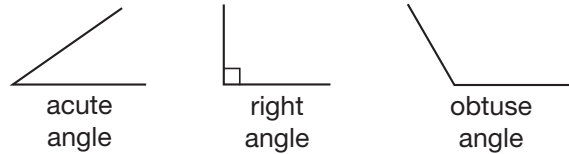
Triangles that contain these angles can be classified as acute, right, or obtuse.

Triangles Classified by Angles

| acute triangle | right triangle | obtuse triangle |
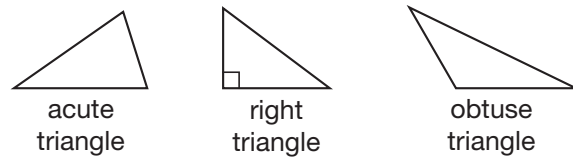
Every triangle has at least two acute angles. If all three angles are acute, the triangle is an **acute triangle.** If one of the angles is a right angle, the triangle is a **right triangle.** If one of the angles is obtuse, the triangle is an **obtuse triangle.**
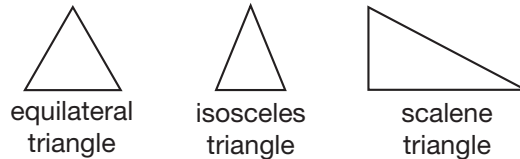
We can also classify triangles by the comparative lengths of their sides.

Triangles Classified by Sides

| equilateral triangle | isosceles triangle | scalene triangle |

Thinking Skill

Classify

Can a right triangle also be an isosceles triangle? Why or why not?

The three sides of an **equilateral triangle** have equal lengths. At least two sides of an **isosceles triangle** have equal lengths. All three sides of a **scalene triangle** have different lengths.

Every triangle can be classified both by angles *and* by sides. Notice that the scalene triangle illustrated above also appears to be a right triangle, while the isosceles and equilateral triangles are also acute triangles.

Discuss Can a triangle have parallel sides? Why or why not?

Example 1

All three sides of this triangle are congruent. Which of the following terms does *not* describe the triangle?
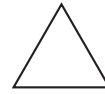
A equilateral   B acute   C isosceles   D right

The triangle is an equilateral triangle and an acute triangle. It is also an isosceles triangle, because *at least* two of the sides have equal lengths. Because none of the angles of the triangle are right angles, the correct answer is **D.**

Example 2

(**Represent**) **Draw three angles with sides that are segments of equal length. Make the first angle acute, the second right, and the third obtuse. Then, for each angle, draw a segment between the endpoints to form a triangle. Classify each triangle by sides and by angles.**

We draw each angle with two segments that have a common endpoint.

Then we draw segments to form three triangles. Since two sides of each triangle have equal length, all three triangles are isosceles. Here are the classifications for each triangle:

isosceles and   **isosceles** and   **isosceles** and
acute   **right**   **obtuse**
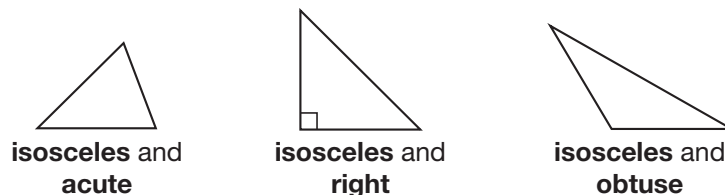
(**Verify**) Which triangle above has perpendicular sides? How do you know?

Lesson Practice   (**Conclude**) Classify each triangle by angles:

a.   b.   c.

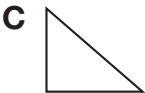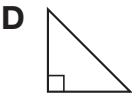(**Conclude**) Classify each triangle by sides:

d.   e.   f.
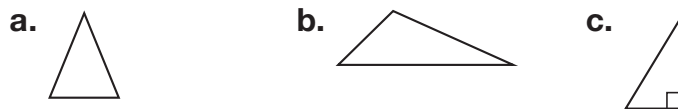
g. (Represent) Draw a right triangle.

h. (Represent) Draw an equilateral triangle.

i. (Represent) Draw a right angle with sides that are segments of equal length. Then draw a segment between the endpoints to form a triangle. What type of right triangle did you draw?

j. (Model) Draw a diagonal segment on a sheet of paper between opposite corners (vertices), dividing the rectangular paper into two congruent triangles. Classify the triangles that are formed by sides and by angles.

Written Practice

Distributed and Integrated

1. (Represent) Draw a pair of horizontal parallel line segments. Make both segments the same length.
(31)

(Formulate) For problems **2–4,** write an equation and find the answer.

*2. The Aerial Drawbridge in Minnesota is 386 feet long. The Roosevelt Island Drawbridge in New York is 418 feet long. How many feet longer is the Roosevelt Island Drawbridge?
(9, 35)

*3. For a field trip, 336 students at an elementary school will be transported by 7 school buses. Each bus will carry the same number of students. How many students will ride on each bus?
(21)

*4. A fortnight is 2 weeks. How many days is a fortnight? Use a multiplication pattern.
(21)

5. Round 780 to the nearest hundred.
(33)

6. Multiple Choice Which triangle has one obtuse angle?
(36)

A   **B**   **C**   **D**

7. How many years were there from 1776 to 1976?
(35)

8. When the students voted for president, Tendai received 119 votes and Juanita received 142 votes. Juanita won by how many votes?
(9, 35)

9. What is the name for the top number of a fraction?
(Inv. 2)

* **10. a. Multiple Choice** Which of these shapes is *not* a polygon?
(32)

A   B   C   D
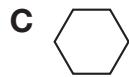
    b. ✎ (Verify) Explain your choice in part **a.**

* **11.** (Analyze) Cindy has two fourths of a circle and three tenths of a circle. What does she need to make a whole circle?
(Inv. 2)

12. 763
(29) $\times$ 800

13. $24.08
(17) $\times$ 6

14. 976
(29) $\times$ 40

15. 400
(29) $\times$ 50

16. 5818
(14) $-$ m
     4747

17. $98.98
(13) $36.25
    $ 4.97
   + $87.64

18. 1010
(9) $-$ 918

19. $7w = \$7.63$
(18, 34)

20. $368 \div 9$
(34)

21. $6\overline{)4248}$
(34)

22. $8\overline{)\$10.00}$
(26)

23. (Conclude) What are the next three terms in this counting sequence?
(1)

     ..., 2700, 2800, 2900, ____, ____, ____, ...

* **24.** (Analyze) What fraction of this hexagon is shaded? Is more than or less than 25% of the hexagon shaded? Is more than or less than 10% of the hexagon shaded?
(30)
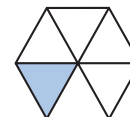
25. (Connect) To what number is the arrow pointing?
(12)

     600   700   800   900   1000
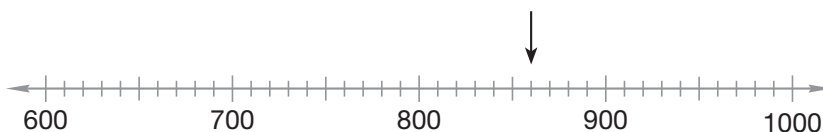
*26. Show how to check this division answer. Is the answer correct?
(26)

$$784 \div 6 = 13 \text{ R } 4$$

27. Write a multiplication fact that shows how to find the
(13) number of small squares in this rectangle.
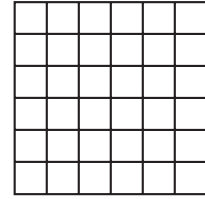
28. Seven tenths of a circle is what percent of a circle?
(Inv. 2)

29. (**Represent**) Draw an isosceles right triangle.
(36)

*30. (**Estimate**) For a short distance the maximum speed that a human
(33) can run is about 23 miles per hour. For a short distance the maximum speed that a pronghorn antelope can run is about 61 miles per hour. About how many miles per hour faster can a pronghorn antelope run?

Early Finishers

Real-World Connection

Copy each upper case letter below onto your paper. Then draw one more segment on each letter to form a triangle. Classify each triangle by its angles and then by its sides.

a. **V**　　　　　　　　　b. **L**
c. **X**　　　　　　　　　d. **Z**

• Drawing Pictures of Fractions

Power Up

facts   Power Up F

count aloud   Count by 12s from 12 to 120.

mental math

  a. Time: How many months are in 4 years? … 5 years? … 6 years?

  b. Estimation: Round 890 to the nearest hundred. Then multiply by 4. What is the answer?

  c. Number Sense: 4×89

  d. Number Sense: 4×90 minus 4×1

  e. Percent: 50% of 60¢

  f. Percent: 25% of 60¢

  g. Percent: 10% of 60¢

  h. Calculation: $9 \times 9, - 9, \div 9$

problem solving   Choose an appropriate problem-solving strategy to solve this problem. The license plates of a certain state have three letters followed by three digits. One license plate reads "CAR 123." How many different license plates from the state could begin with "CAR" and end with any arrangement of all the digits 1, 2, and 3? List the possible license plates.
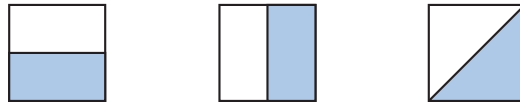
New Concept

A picture can help us understand the meaning of a fraction.

Example 1

Draw three squares and shade $\frac{1}{2}$ of each square a different way.

The denominator of $\frac{1}{2}$ tells us to cut each square into two equal parts. The numerator of the fraction tells us to shade one of the parts. There are many ways to do this. Here we show three different ways:

When drawing pictures of fractions, we must always be careful to divide the pictures into *equal* parts. The square below has been cut into two parts, but the parts are not equal. Therefore, the square has not been divided into halves.
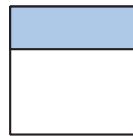
  This does not represent $\frac{1}{2}$.

Thinking Skill

Model

How many different ways can you fold a piece of paper to show $\frac{1}{4}$?

Example 2

Draw a rectangle and shade $\frac{1}{3}$ of it.

After we draw the rectangle, we must divide it into three equal parts. If we begin by dividing it in half, we will not be able to divide it into three equal parts.
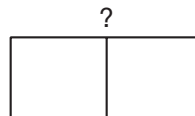
We must plan ahead. To form 3 parts, we draw 2 equally spaced segments. Here we show two different ways to shade $\frac{1}{3}$ of a rectangle:

Example 3

A circular courtyard is made of white stone and blue stone. If $\frac{1}{2}$ of the courtyard is made of blue stone, which diagram could *not* represent the courtyard? Explain why.

A B C D

We see that the circular courtyard is divided into eight equal parts. Since 4 is $\frac{1}{2}$ of 8 and 4 parts are made of blue stone, then half of the courtyard is blue stone. Diagrams **A, C,** and **D** all show that half of the courtyard is blue. Diagram **B** could not represent the courtyard because only 2 of the 8 sections are blue.

Lesson Practice

a. **Represent** Draw a circle and shade $\frac{1}{4}$ of it.

b. **Represent** Draw a rectangle and shade $\frac{2}{3}$ of it.

c. **Represent** The shaded portion of this square represents the fraction $\frac{3}{4}$. Show another way to shade $\frac{3}{4}$ of a square.

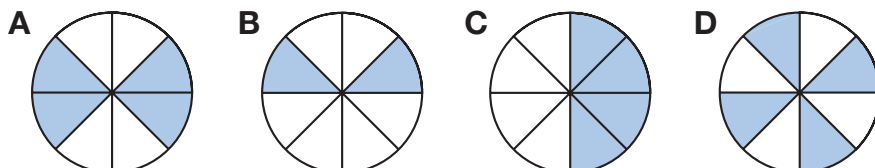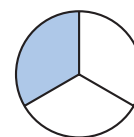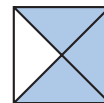d. **Represent** The shaded portion of this circle represents the fraction $\frac{1}{3}$. Draw a circle and shade $\frac{2}{3}$ of it.

Written Practice   *Distributed and Integrated*

*1. **Represent** Draw a pair of horizontal parallel line segments. Make the
(12, 31) lower segment longer than the upper segment.

2. **Represent** Draw three rectangles and shade $\frac{1}{2}$ of each rectangle a
(30, 37) different way. What percent of each rectangle is shaded?

Formulate For problems **3–5,** write an equation and find the answer.

3. When Ramon cleaned his room, he found 39 marbles, 20 baseball
(11) cards, a toothbrush, 4 pencils, and a sock. How many items did he find?

4. A foot is equal to 12 inches. How many inches does 3 feet equal?
(17, 21)

5. How many years were there from 1517 to 1620?
(35)

6. **List** Write the factors of 40.
(25)

*7. What fraction of this octagon is not shaded? Is more than
(30) or less than 50% of the octagon not shaded? What percent of the octagon is shaded?
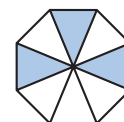
8. Joleen awoke this morning at 6:37 a.m. and left for school 48 minutes
(35) later. What time did Joleen leave her home to go to school?

9. Round 46 to the nearest ten.
(33)

* 10. (**Represent**) Draw a right triangle.
(36)

11. $36.51   **12.**   3040   **13.** $90.00
(13) $74.15   *(14)* $-$ w *(13)* $-$ $20.30
    $+$ $25.94   2950

14.   592   **15.** $4.75   **16.**   43
(29) $\times$ 90   *(29)* $\times$ 80   *(10)* c
                                                      29
                                                   467
                                             $+$ 94
                                                   700

17. Compare: $\dfrac{840}{8}$ ◯ $\dfrac{460}{4}$ **18.** 720×400
(Inv. 2)   *(29)*

19. $6w = \$12.24$ **20.** $1000 \div (100 \div 10)$
(26, 34)   *(24, 34)*

21. $60 \times (235 \div 5)$ **22.** $42 \times 30 \times 7$
(24, 26)   *(18, 29)*

23. $\$20 - (\$3.48 + \$12 + \$4.39)$
(13, 24)

24. (**Explain**) Duncan fit one half of a circle, one fourth of a circle, and
(Inv. 2) one tenth of a circle together. What percent of the circle was missing?
Explain how you found your answer.

* 25. Multiple Choice Which two of these shapes are *not* polygons?
(32)   **A**   **B**   **C**   **D**

26. (**Represent**) Draw a rectangle and shade $\frac{1}{3}$ of it.
(37)

27. What year was one decade before 1932?
(28)

*28. a. (**Verify**) An octagon has how many angles?
(23, 32)

    b. (**Verify**) A quadrilateral has how many angles?

    c. Use the answers to parts **a** and **b** to write a fraction equal to $\frac{1}{2}$.

29. Multiple Choice Which of these shapes is *not* a quadrilateral?
(32)

A

B Elm Ave.

C MATH

D YIELD
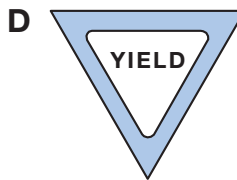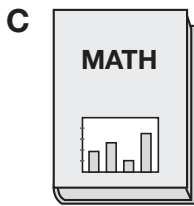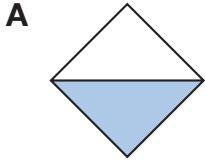
30. (**Estimate**) The highest-scoring All-Star game in the National Basketball
(33) Association happened in 1987 when the West team defeated the East
team 154–149. About how many points were scored during that game
altogether?

Early Finishers

Real-World Connection

Monique has 64 songs on her mp3 player. Three eighths of the songs
are less than three minutes long.

    a. Draw a diagram to illustrate the problem.

    b. Use the diagram to find the number of songs that are
less than three minutes long.

• Fractions and Mixed Numbers on a Number Line

Power Up

facts	Power Up D
count aloud	Count by 7s from 7 to 84.

mental math

a. **Number Sense:** Le purchased 5 dozen eggs. How many eggs is that?

b. **Estimation:** Round 615 to the nearest hundred. Then add 800. What is the answer?

c. **Number Sense:** One CD case weighs 70 grams. Ten CD cases weigh how many grams?

d. **Time:** Twenty-four hours is one day. How many hours is 5 days?

e. **Percent:** 50% of 80¢

f. **Percent:** 25% of 80¢

g. **Percent:** 10% of 80¢

h. **Calculation:** $6 \times 6, -6, \div 6, +1, -6$

problem solving

Choose an appropriate problem-solving strategy to solve this problem. Belinda wrote a multiplication problem and then erased one of the factors and one of the digits in the product. She gave it to Laurel as a problem-solving exercise. Copy Belinda's multiplication problem and find the missing digits for Laurel.

$$\begin{array}{r} \_\_ \\ \times\ \ 8 \\ \hline \_6 \end{array}$$

A number line is made up of a series of points. The points on the line represent numbers. On the following number line, whole numbers are labeled. However, there are many numbers on the line that are not labeled. We mark some unlabeled numbers with arrows:

Many of the unlabeled points on a number line can be named with fractions and **mixed numbers.** A mixed number is a whole number and a fraction together, like $1\frac{1}{2}$ (one and one half), for example.

To identify a fraction or mixed number on a number line, we need to count the divisions between the whole numbers. On the number line below, the distance between every two whole numbers has been divided into three sections (or into thirds). Thus, each small section is one third ($\frac{1}{3}$). (Be careful to count the *sections* of the number line and not the marks that separate the sections.)
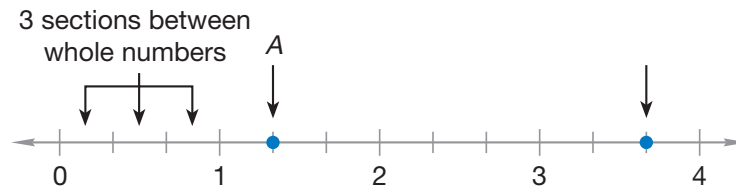
A point on a number line is named by its distance from zero. The location of the point marked by arrow *A* is given by the whole number 1 plus the length of one section. The number for that point is $1\frac{1}{3}$. The point marked with arrow *B* is the whole number 3 plus the length of two sections. The number for point *B* is $3\frac{2}{3}$.

When reading from number lines with sections smaller than 1, follow these steps:

Step 1: Find the whole-number distance from zero up to (but not past) the point to be named. This is the whole-number part of the answer.

Step 2: Count the number of sections between whole numbers. This number is the denominator of the fraction.

Step 3: Then count the number of sections past the whole number to the point being named. This is the numerator of the fraction.

Example 1

Name the fraction or mixed number marked by each arrow on these number lines:
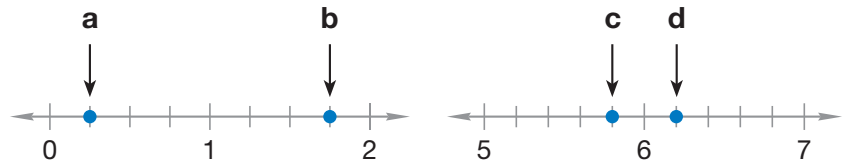
Point **a** is between 0 and 1, so it is named by a fraction and not by a mixed number. The distance between whole numbers on this number line is divided into fourths. Point **a** is one section from zero, which is $\frac{1}{4}$.

The distance from zero to point **b** is 1 plus the length of three sections, or **$1\frac{3}{4}$**.

The distance from zero to point **c** is 5 plus a fraction. The distance between whole numbers on this number line is divided into fifths. Point **c** is four sections from 5, which is **$5\frac{4}{5}$**.

The distance from zero to point **d** is 6 plus the length of one section, or **$6\frac{1}{5}$**.

Example 2

Ra'Shawn walked $\frac{2}{3}$ of a mile. D'Neese walked $\frac{3}{4}$ of a mile. Who walked farther?

Here we show two number lines. On one number line the fraction $\frac{2}{3}$ is graphed. On the other number line $\frac{3}{4}$ is graphed.
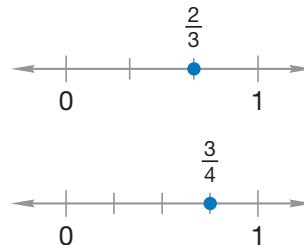
Refer to these number lines to compare the fractions $\frac{2}{3}$ and $\frac{3}{4}$.

$$\frac{2}{3} \bigcirc \frac{3}{4}$$

Both $\frac{2}{3}$ and $\frac{3}{4}$ are greater than 0 but less than 1. Since numbers to the right on the number line are greater than numbers to the left, we see that $\frac{3}{4}$ is greater than $\frac{2}{3}$.

$$\frac{2}{3} < \frac{3}{4}$$

D'Neese walked farther than Ra'Shawn.

Lesson Practice **Connect** Name the fraction or mixed number marked by each arrow on these number lines:

a. **b.** **c.** **d.**

Three fractions are graphed on the number line below. Refer to the number line to compare the fractions in problems **e–g.**
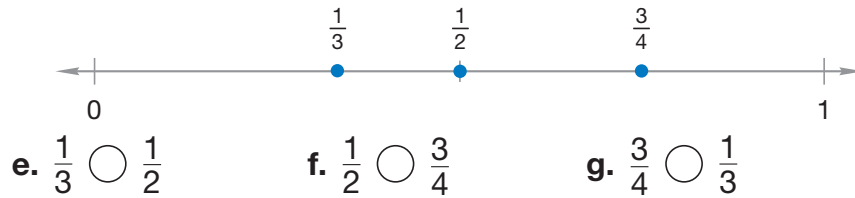
$$\frac{1}{3} \qquad \frac{1}{2} \qquad \frac{3}{4}$$

0   1
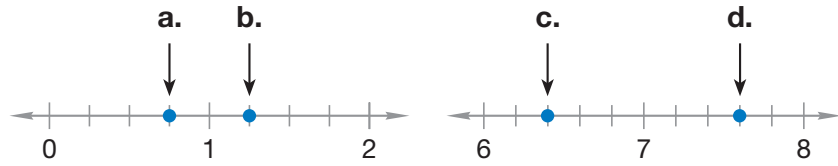
e. $\frac{1}{3} \bigcirc \frac{1}{2}$ **f.** $\frac{1}{2} \bigcirc \frac{3}{4}$ **g.** $\frac{3}{4} \bigcirc \frac{1}{3}$

Written Practice
Distributed and Integrated

*1. **Represent** Draw a pair of horizontal parallel line segments. Make
(31) the upper segment longer than the lower segment.

2. Kione scored $\frac{1}{4}$ of the team's 28 points. How many points did Kione
(Inv. 2) score?

Formulate For problems **3–6,** write an equation and find the answer.

3. Tickets to the matinee were $15 each. Mr. Rodriquez bought
(21) four tickets. What was the total cost of the tickets?

4. The used-car dealer bought a car for $725 and sold it for $1020.
(35) How much profit did the dealer make on the car? Use a subtraction
formula.

5. In two hours the 3 boys picked a total of 1347 cherries. If they share
(21) the cherries evenly, then each boy will get how many cherries?

6. How many years were there from 1950 to 1989?
(35)

*7. **Multiple Choice** Which triangle has three acute angles?
(36)

A   **B**   **C**   **D**
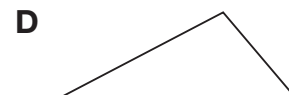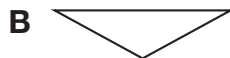
8. **Represent** Draw a circle and shade $\frac{3}{4}$ of it. What decimal part of
(30, 37) the circle is shaded?

*9. How many days are in a leap year?
(28)

10. A stop sign has the shape of an octagon. An octagon has how many
(32) sides?

11. 3647 + 92 + 429   **12.** 3518 − 1853
(6)   (9)

*13. 4 × 6 × 8 × 0   **14.** 3518 ÷ 10
(15)   (26)

15. $4.76 + $12 + $0.97 + w = $20
(10, 13)

16. $100 − $87.23   **17.** 786 × 900
(13)   (29)

18. $63.18 ÷ 9   **19.** 375 × (640 ÷ 8)
(34)   (24, 29)

*20. Compare: (3 × 5) × 7 ◯ 3 × (5 × 7)
(24)

*21. **Multiple Choice** Every four-sided polygon is which of the
(32) following?

    A square   **B** rectangle   **C** quadrilateral   **D** rhombus

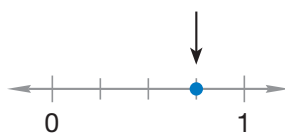**22.** **Conclude** What are the next three terms in this counting sequence?
(1)

                  ..., 1800, 1900, 2000, ＿＿, ＿＿, ＿＿, ...

23. **Connect** To what mixed number is the arrow pointing?
(38)
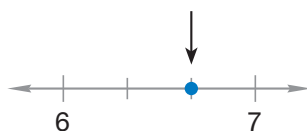
24. **Connect** To what fraction is the arrow pointing?
(38)

25. It is 9:45 a.m. What time will it be in 4 hours?
(28)

26. Round 649 to the nearest hundred.
(33)

27. If the divisor is 6 and the quotient is 3, then what is the dividend?
(20)

*28. (**Conclude**) **a.** Are all squares congruent?
(32)

      b. Are all squares similar?

29. Refer to the number line below to complete the comparison:
(38)

$$\frac{1}{2} \bigcirc \frac{1}{6}$$
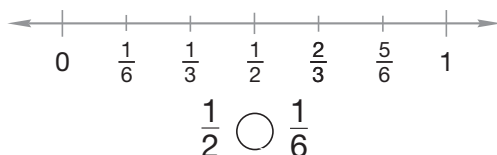
30. Use data from the table to answer the questions that follow.
(9, 33)

Tall Buildings in North America

Building	Location	Stories
Two Liberty Place	Philadelphia, PA	58
One Post Office Square	Boston, MA	40
Water Tower Place	Chicago, IL	74
Calpine Center	Houston, TX	34
101 Montgomery Street	San Francisco, CA	29

    a. Which two buildings have a combined height that is the same as
(6)   Water Tower Place?

    b. How many stories taller is Two Liberty Place than 101 Montgomery Street?
(9)

    c. (**Justify**) Explain how an estimate can be used to compare the height of
(33)   Two Liberty Place to the height of 101 Montgomery Street.

Early Finishers
Real-World Connection

Ted ran $1\frac{1}{3}$ miles. Describe how to draw a number line from 0 to 2 that shows $1\frac{1}{3}$. Then draw the number line, and label $1\frac{1}{3}$.

• Comparing Fractions by Drawing Pictures

Power Up

facts Power Up F

count aloud Count by 7s from 7 to 84.

mental math

a. **Time:** How many days is 3 weeks? … 4 weeks? … 6 weeks?

b. **Estimation:** Round 78 to the nearest ten. Then add 70. What is the answer?

c. **Number Sense:** Apria had 830 pennies in her piggy bank. She put 200 of the pennies into coin rolls. How many pennies are left in her piggy bank?

d. **Time:** How many hours is 6 days? (*Think:* 6×24.)

e. **Fractional Parts:** $\frac{1}{2}$ of 44 pounds

f. **Percent:** 50% of $2.00

g. **Percent:** 25% of $2.00

h. **Calculation:** $5 \times 5, + 5, \div 5, - 5$

problem solving Choose an appropriate problem-solving strategy to solve this problem. Lu'Ann is working through a book of 40 crossword puzzles. The puzzles are divided into five units according to difficulty, and each unit contains 8 puzzles. Lu'Ann began with the first puzzle of the book and is solving each puzzle in order until she finishes the book. She completes 6 puzzles each day. On which day will Lu'Ann's sixth puzzle for the day be the final puzzle in one of the book units? How many puzzles will she have completed at that point?

One fourth of the circle on the left is shaded. One half of the circle on the right is shaded.
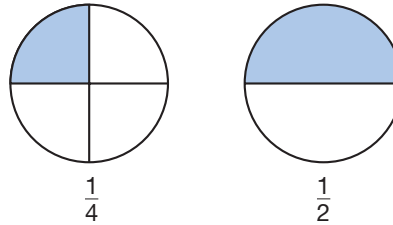
$$\frac{1}{4} \qquad \frac{1}{2}$$

We see that less of the circle on the left is shaded. This is because $\frac{1}{4}$ is a smaller fraction than $\frac{1}{2}$. We can write this comparison using a comparison symbol as

$$\frac{1}{4} < \frac{1}{2}$$

In this lesson we will begin comparing fractions by drawing pictures of the fractions and comparing the pictures.

Example

Armando finished his math assignment in $\frac{1}{3}$ of an hour. Jan finished the assignment in $\frac{1}{2}$ of an hour. Who took longer to finish the assignment? Draw pictures to compare these fractions: $\frac{1}{2} \bigcirc \frac{1}{3}$

We might think that $\frac{1}{3}$ is greater than $\frac{1}{2}$ because 3 is greater than 2. However, by drawing pictures, we will see that $\frac{1}{3}$ is actually less than $\frac{1}{2}$. If an object is divided into 3 parts, each part will be smaller than if the object were divided into 2 parts.

To begin, we draw two *congruent* shapes. We draw two equal-sized rectangles, and we label the rectangles $\frac{1}{2}$ and $\frac{1}{3}$. Next, we divide the rectangles into the number of parts shown by the denominator, and we shade the number of parts shown by the numerator.

$\frac{1}{2}$

$\frac{1}{3}$
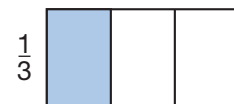
Then we compare the shaded areas. We see that more of the rectangle is shaded when $\frac{1}{2}$ is shaded than when $\frac{1}{3}$ is shaded.

$$\frac{1}{2} > \frac{1}{3}$$

Since $\frac{1}{2}$ is greater than $\frac{1}{3}$, $\frac{1}{2}$ of an hour is greater than $\frac{1}{3}$ of an hour, so **Jan took longer** to finish the assignment.

Generalize These fractions are written in order from least to greatest:

$$\frac{1}{9}, \frac{1}{8}, \frac{1}{7}, \frac{1}{6}, \frac{1}{5}, \frac{1}{4}, \frac{1}{3}, \frac{1}{2}$$

What conclusion can be made about the values of fractions when the numerators are the same?

Lesson Practice   **Represent** Draw pictures to compare each pair of fractions. When drawing pictures of any two fractions, be sure to draw the shapes the same size.

a. $\frac{1}{2} \bigcirc \frac{2}{3}$ **b.** $\frac{1}{2} \bigcirc \frac{2}{4}$

c. $\frac{1}{3} \bigcirc \frac{1}{4}$ **d.** $\frac{2}{3} \bigcirc \frac{3}{4}$

Written Practice

Distributed and Integrated

*1. **Represent** Draw a pair of horizontal parallel line segments of the same length. Form a quadrilateral by connecting the ends of the segments.
(31, 32)

Formulate For problems **2–4,** write an equation and find the answer.

2. How many years is five centuries?
(21, 28)

3. Paloma is 6 years older than her sister. If Paloma is 13 years old, then how old is her sister? Use a subtraction pattern.
(35)

4. Diego walked 488 feet going to the end of the pier and back. How long is the pier?
(21)

5. **Represent** Draw pictures to compare these fractions: $\frac{1}{4} \bigcirc \frac{1}{3}$
(39)

6. What number is half of 23?
(2)

7. Emily's cat ate $\frac{1}{4}$ of a dozen fish fillets. How many fish fillets did Emily's cat eat?
(Inv. 2)

8. Round 84 to the nearest ten.
(33)

*9. **(List)** Write the factors of 35.
(25)

10. $93.18
(13) $42.87
   + $67.95

11. $30.00*
(13)   − $ 8.75

*12.   46
(6)   23
    97
    15
    24
    55
  + 55

13.   4304
(14)   − b
    3452

14. $6.38
(29)   ×   60

15.   640
(29)   ×   700

16. $\dfrac{640}{8}$
(34)

17. $\dfrac{720}{10}$
(26)

18. $\dfrac{\$6.24}{6}$
(34)

19. $\dfrac{1236}{4}$
(34)

20. $563 \div 7$
(34)

21. $4718 \div 9$
(26)

22. $8m = 3000$
(26)

23. The latest showing of a movie ends at 20 minutes before midnight.
(28)   At what time does the movie end?

*24. A quarter of a circle plus an eighth of a circle is what percent of a circle?
(Inv. 2, Inv. 3)

25. According to this calendar, what was the date of the
(28)   third Saturday in April 1901?

APRIL 1901						
S	M	T	W	T	F	
	1	2	3	4	5	6
7	8		10	11	12	13
14	15	16	17	18	1	20
21	22	23	24	25	26	27
28	2	30				

*26. **(Connect)** Refer to the number line below to answer parts **a–c.**
(38)
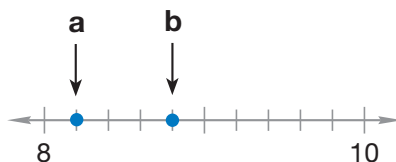
a. To what mixed number is arrow **a** pointing?

b. To what mixed number is arrow **b** pointing?

c. Write your answers to **a** and **b** using a comparison symbol to show which mixed number is greater and which is less.

27. What is the product of four hundred sixteen and sixty?
(5, 29)

28. **a.** How many hours are in a day?
(23, 28)

    b. How many hours are in half a day?

    c. Use your answers to parts **a** and **b** to write a fraction equal to $\frac{1}{2}$.

29. D'Arla works each day from 7:45 a.m. to 4:15 p.m. During that time,
(35) she is not paid for a 45-minute lunch break. How many hours is D'Arla paid for each day she works?

30. ✏️ **Estimate** A group of friends are planning to spend 7 days hiking
(33) a 135-mile portion of the Appalachian Trail and expect to hike about the same number of miles each day. Estimate the number of miles the friends plan to hike each day. Explain your answer.

Early Finishers

Real-World Connection

Seth and Margie each ordered a 12-slice veggie pizza for dinner. Seth ate $\frac{1}{2}$ of his pizza. Margie ate $\frac{1}{3}$ of her pizza.

    a. Draw two circles. Then shade one circle to represent the amount of pizza that Seth ate, and shade one circle to represent the amount of pizza that Margie ate.

    b. Which is greater: $\frac{1}{2}$ or $\frac{1}{3}$? Write a comparison using the fractions and a comparison symbol.

    c. Find the number of slices of pizza Seth ate and the number of slices that Margie ate.

• Writing Quotients with Mixed Numbers

Power Up

facts   Power Up E

mental math

   a. Time: What time is 22 minutes after 3:15 p.m.?

   b. Number Sense: 600×4

   c. Measurement: Twelve inches is one foot. How many inches is half a foot? … one and a half feet? … two and a half feet?

   d. Fractional Parts: $\frac{1}{2}$ of 21

   e. Money: $10 \times 100¢$

   f. Percent: What is 50% of $20.00?

   g. Percent: The shirt is on sale for 25% off the regular price of $20.00. How much is 25% of $20.00?

   h. Calculation: $4 \times 2, + 1, \times 3, + 3, \div 3, \div 2$

problem solving

In bowling, a *spare* occurs when two rolls are used to knock down all 10 pins. Knocking down 3 pins on the first roll and 7 pins on the second roll is one way to bowl a spare. What are all the possible ways to get a spare in bowling?

⑦ ⑧ ⑨ ⑩
④ ⑤ ⑥
② ③
①
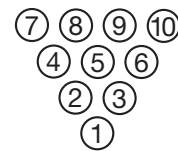
Focus Strategy: Make or Use a Table, Chart, or Graph

(**Understand**) We are told that a spare in bowling occurs when two rolls are used to knock down all 10 pins. We are given an example of one of the ways to make a spare. We are asked to find all the possible ways to get a spare.

Plan We will make a table to help us find all the possible ways to make a spare. A table will help us organize our answer and ensure that we find all the possible ways to make a spare.

Solve We create two columns for our table and label the columns "1st roll" and "2nd roll." We start filling in the table by writing what happens if 0 pins are knocked down on the first roll. We write "0" in the first column. For a spare to occur, all ten pins must be knocked down, so 0 on the first roll means 10 pins are knocked down on the second roll. We write "10" in the second column.

Next, we record what happens when 1 pin is knocked down on the first roll. Nine pins would remain to be knocked down on the second roll, so we write "1" in the first column and "9" in the second column.

We continue to add rows to the table until we reach 9 pins on the first roll and 1 pin on the second roll.

1st roll	2nd roll
0	10
1	9
2	8
3	7
4	6
5	5
6	4
7	3
8	2
9	1

Check We know that our answer is reasonable because we found two numbers that total 10 for each possible first roll of the ball. We did not include a row in the table for 10 pins on the first roll. Knocking down 10 pins on the first roll—this called a *strike*—means that all the pins are knocked down, and there is no need for a second roll. Making a table helped us find all the possibilities.
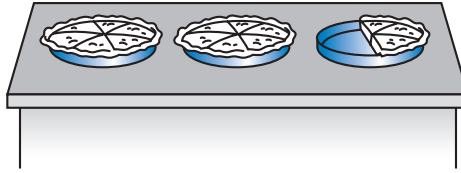
Reading Math

A mixed number can represent the sum of whole numbers and a fraction.

$1 + 1 + \frac{1}{2} =$
$2 + \frac{1}{2} = 2\frac{1}{2}$

The picture below shows some potpies on a shelf.

We see two whole potpies and one half of another potpie. There are two and one half potpies on the shelf. Using digits, we write "two and one half" this way:

$$2\frac{1}{2}$$

Example 1

Use a mixed number to name the number of shaded circles shown here.
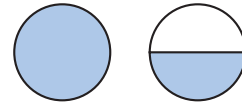
We see two circles. The completely shaded circle represents the whole number 1. Half of the second circle is shaded. It represents the fraction $\frac{1}{2}$. Together, the number of shaded circles is one and one half.

$$1\frac{1}{2}$$

Some problems have answers that are mixed numbers. For example, what is the width of the rectangle formed by folding a sheet of notebook paper in half?

Unfolded

11 n.

$8\frac{1}{2}$ n.

Folded

?

$8\frac{1}{2}$ n.
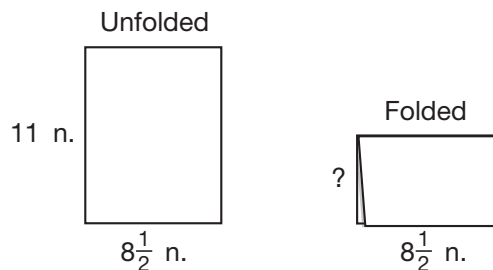
The width of the rectangle is half of 11 inches, which is $5\frac{1}{2}$ inches.

Example 2

Paco, Alejandro, and Lucy will equally share seven chicken potpies. How many pies are there for each person?

First we will use a diagram to explain the solution. We need to divide the potpies into three equal groups. We can arrange six of the potpies into three groups of two potpies.

2 for Paco   2 for Alejandro   2 for Lucy

However, there are seven potpies, so there is still one potpie to be divided. We divide the remaining potpie into thirds:
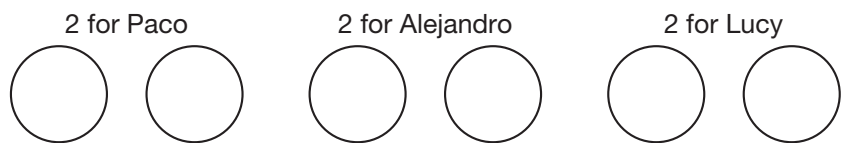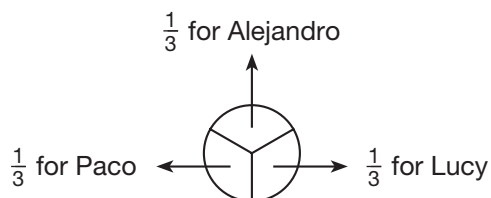

$\frac{1}{3}$ for Alejandro

$\frac{1}{3}$ for Paco $\frac{1}{3}$ for Lucy

We find that there are $2\frac{1}{3}$ **potpies** for each person.

Now we will show how to find the answer using a pencil-and-paper algorithm. To divide seven potpies into three equal groups, we divide 7 by 3.

$$3\overline{)7} \quad \begin{array}{r} 2 \\ -6 \\ \hline 1 \end{array}$$

The quotient is 2, which means "2 whole potpies." The remainder is 1, which means one potpie has not been divided. Now we divide the remaining potpie by three.

One divided by three is the fraction one third. We write "$\frac{1}{3}$" after the whole number above the division box.

$$3\overline{)7} \quad \begin{array}{r} 2\frac{1}{3} \\ -6 \\ \hline 1 \end{array}$$

This answer means that each person will get $2\frac{1}{3}$ **potpies.**

Math Language

The remainder of a division problem is written as the numerator of a fraction that has the divisor as its denominator.

Example 3

A grocer divided a 5-pound block of mozzarella cheese into 4 equal pieces. The pieces were then wrapped and displayed for sale. Which quotient represents the weight in pounds of each piece?
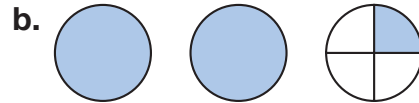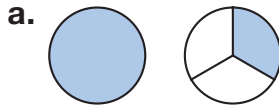
$$
\begin{array}{r}
1 \text{ R } 1 \\
4\overline{)5} \\
-4 \\
\hline
1
\end{array}
\qquad
\begin{array}{r}
1\tfrac{1}{4} \\
4\overline{)5} \\
-4 \\
\hline
1
\end{array}
$$

It is possible to divide the cheese into 4 pieces of equal weight, so the weight of each piece is **$1\tfrac{1}{4}$ pounds.**

Lesson Practice

Write a mixed number to name the number of shaded circles in each diagram:

a.

b.

Represent Draw and shade circles to represent each mixed number:

c. three and one half

d. one and three fourths

e. **Represent** Use a diagram to explain the answer to the problem below. Then show how to find the answer using pencil and paper.

Taro, Shasa, Layne, and Cynthia will equally share nine chicken potpies. How many pies are there for each person?

Written Practice

Distributed and Integrated

*1. **Represent** Draw a pair of horizontal parallel line segments. Make the
(31, 32) lower segment longer than the upper segment. Connect the endpoints of the segments to form a quadrilateral.

2. If 1 pizza is shared equally by 6 people, then each person will get what
(37) fraction of the pizza?

3. Hikaru, Luz, and Obi are sharing 4 oranges equally. How many oranges
(40) does each person have?

4. How many circles are shaded?
(40)
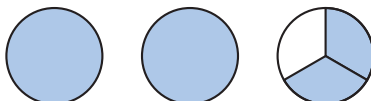
Formulate For problems **5–7,** write an equation and find the answer.

*5. One hundred forty students were divided equally into 5 classes. How
(21) many students were in each class?

6. A veterinarian measured the weight of Khanh's two golden retrievers as
(35) 69 pounds and 83 pounds. How many fewer pounds did Khanh's
69-pound golden retriever weigh?

7. The first flag of the United States had 13 stars. How many more stars
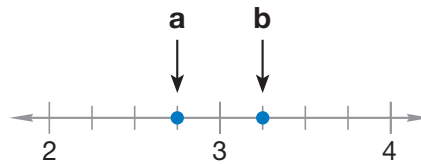(35) does the current flag have?

8. A hexagon has how many more sides than a pentagon?
(32)

9. One half of a circle plus one fourth of a circle is what percent of a whole
(Inv. 2) circle?

*10. **Connect** Refer to the number line below to answer parts **a–c.**
(38)

a   b

2   3   4

a. To what mixed number is arrow **a** pointing?

b. To what mixed number is arrow **b** pointing?

c. Write your answers to **a** and **b** using a comparison symbol to show
which number is greater and which is less.

11. What percent is half of 25%?   **12.** $m - 345 = 534$
(2,   (14)
Inv. 3)

13.   785   **14.**   7106   **15.** $3.84
(6)   964   (9) $- 3754$ (29) $\times$ 60
          287
        $+ 846$

16. 769×800 **17.** $\dfrac{\$24.48}{8}$ **18.** $\dfrac{4320}{9}$
(29)   (34)   (34)

19. $20 − ($1.45 + $6.23 + $8)   **20.** 3742 + 3742 + 3742 + 3742 + 3742
(13, 24)   (13, 17)

21. Round 650 to the nearest hundred.
(33)

22. A year is what fraction of a decade? A year is what percent of a
(28) decade?

23. Multiple Choice Which of these angles appears to be an obtuse
(31) angle?

A   **B**   **C**   **D**

24. (Conclude) What are the next three terms in this counting sequence?
(1)

$$\ldots, 60, 70, 80, \underline{\quad}, \underline{\quad}, \underline{\quad}, \ldots$$

25. On April 2, the low temperature of the day in Madison, Wisconsin, was
(27) 48°F. The high temperature of the day was 13°F higher. What was the
high temperature that day?

26. (Represent) Draw two circles of the same size. Shade $\frac{1}{4}$ of one circle
(39) and $\frac{1}{3}$ of the other circle. Then compare these fractions:

$$\frac{1}{4} \bigcirc \frac{1}{3}$$

*27. Multiple Choice Which of these fractions is greater than one half?
(23)
   A $\frac{5}{12}$ **B** $\frac{3}{5}$ **C** $\frac{7}{14}$ **D** $\frac{10}{21}$

28. At a bus stop, the first three scheduled bus arrival times of the day are
(35) 6:42 a.m., 7:17 a.m., and 7:52 a.m. If that pattern continues, what is the
next scheduled arrival time for the bus?

29. (Estimate) A marathon is a long-distance running event. Steve has
(1) run in four Boston Marathons. The distance of each marathon was
26 miles 385 yards. Explain how to estimate the total number of miles
Steve has run in Boston Marathons.

30. (Estimate) A soccer coach conducts 6 different drills during the first
(33) 55 minutes of each soccer practice. Explain how to estimate the length
in minutes of each drill if the coach spends about the same number
of minutes conducting each drill.

Focus on

• Pattern Recognition

In Lesson 1 we introduced sequences as counting patterns that continue indefinitely. In each of the examples we looked at, we either counted up or down by a fixed amount. But there are many other possible patterns that can determine the terms of a sequence. Rather than adding or subtracting a fixed number, we could also multiply by a fixed number to produce the next terms.

Example 1

What rule seems to describe this sequence? Find the next two terms in the sequence.

$$3, 6, 12, 24, \underline{\quad}, \underline{\quad}, \ldots$$

Since $3 \times 2 = 6$, $6 \times 2 = 12$, and $12 \times 2 = 24$, we **multiply by 2** to find the next terms. Thus, the fifth term of the sequence is 24×2, or **48**, and the sixth term is 48×2, or **96**.

A sequence that counts up (adds) or counts down (subtracts) by the same amount is called an **arithmetic sequence.** A sequence that increases by multiplying by the same number or decreases by dividing by the same number is called a **geometric sequence.**

In problems **1–4** below, decide whether the sequence is arithmetic or geometric. Then write the next three terms.

1. 43, 49, 55, 61, ____, ____, ____, …

2. 2, 4, 8, 16, ____, ____, ____, …

3. 50, 48, 46, 44, ____, ____, ____, …

4. 2, 6, 18, 54, ____, ____, ____, …

5. Samantha has saved $55. Each month she plans to add $8 to her savings. If she does not spend any of the money she saves, how much will Samantha have after one month? Two months? Three months? What kind of sequence are we making?

6. The fruit fly population in an experiment doubled every week. If the experiment began with 50 fruit flies, what was the population after one week? Two weeks? Three weeks? What kind of sequence are we making?

Another kind of pattern found in sequences is *repetition.* This means that the terms of the sequence repeat themselves.

Example 2

List the next three terms.

$$\textbf{4, 5, 8, 4, 5, 8, 4, 5, \underline{\quad}, \underline{\quad}, \underline{\quad}, \dots}$$

In this sequence, the unit "4, 5, 8" appears to be repeating. Since the last term given is 5, the next three terms would be **8, 4, 5.**

Science has demonstrated that the human brain searches for patterns in events and objects. Since the sun rises every day in the east and sets in the west, we expect the same to occur tomorrow. If we see a pattern in a section of floor tiles, we assume that the pattern continues over the whole floor. We might make similar assumptions about sequences. However, if the part of the sequence we are looking at is not large enough, we might assume a pattern that is not actually there. For example, if we see that a sequence begins with

$$4, 6, 4, 6, \dots$$

we might assume that the sequence repeats after every two terms and that the alternating 4s and 6s will continue. However, those terms are also the beginning of the sequence

$$4, 6, 4, 6, 2, 4, 6, 4, 6, 2, \dots$$

which seems to fit a very different pattern. Without clear information about the structure of a sequence, we must be aware that the patterns we see might not really be there.

7. Assuming the following sequence repeats after every three terms, write the next three terms:

$$4, 5, 9, 4, 5, 9, 4, \underline{\quad}, \underline{\quad}, \underline{\quad}, \dots$$

8. Assuming the following sequence repeats after every four terms, write the next three terms:

$$5, 2, 3, 6, 5, 2, \underline{\quad}, \underline{\quad}, \underline{\quad}, \dots$$

9. Assuming the following sequence repeats after every four terms, write the next three terms:

$$B, U, L, B, \underline{\quad}, \underline{\quad}, \underline{\quad}, \dots$$

10. Assuming the sequence in problem **9** repeats after every three terms, write the next three terms.
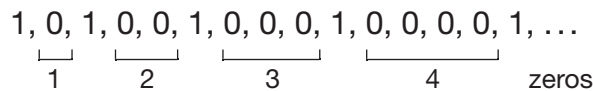
There are many types of patterns that sequences can follow. In the next example, we look at another kind of pattern.

Example 3

What pattern does this sequence appear to follow?

1, 0, 1, 0, 0, 1, 0, 0, 0, 1, …

This sequence of 0s and 1s has 1s separated by an increasing number of 0s. First, one 0 separates 1s; then two 0s; then three 0s. It is reasonable to predict that there will be four more 0s before the next 1 that appears in the sequence.

1, 0, 1, 0, 0, 1, 0, 0, 0, 1, 0, 0, 0, 0, 1, …
　　1　　2　　　3　　　　4　　　zeros

We can predict that there will then be five more 0s between 1s, six more 0s, and so on.

For problems **11–15,** describe the pattern that the sequence appears to follow. Then write the next few terms that seem to fit the pattern.

11. 1, 1, 2, 2, 3, 3, …

12. 0, 2, 0, 4, 0, 6, 0, …

13. A, B, D, E, G, H, …

14. ⊤, ⊣, ⊥, ⊢, ⊤, …

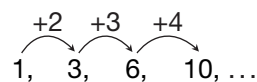**15.** 1, 2, 1, 2, 3, 1, 2, 3, 4, …

Some patterns can be seen more easily by recording the increase or decrease between terms.

Example 4
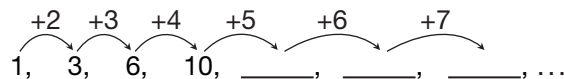
What are the next three terms in this sequence?

0, 1, 3, 6, 10, …

We first find the difference between successive terms.

$$\overset{+2}{\frown}\ \overset{+3}{\frown}\ \overset{+4}{\frown}$$
1,　3,　6,　10, …

The increasing difference from one term to the next also forms a sequence. This sequence may be continued.

$$\overset{+2}{\frown}\ \overset{+3}{\frown}\ \overset{+4}{\frown}\ \overset{+5}{\frown}\ \overset{+6}{\frown}\ \overset{+7}{\frown}$$
1,　3,　6,　10, ____, ____, ____, …

We add 5 to 10 and get 15 for the next term. We add 6 to 15 and get 21 for the following term. We add 7 to 21 and get 28. We have found the next three terms.

1, 3, 6, 10, __15__, __21__, __28__, …

Find the next three terms in these sequences:

16. 1, 4, 9, 16, 25, ____, ____, ____, …

17. 2, 3, 5, 8, 12, ____, ____, ____, …

Example 5

Suppose the first two terms of a sequence are 3 and 4 and that we always get the next term by adding the previous two terms together. The third term would be 3 + 4 = 7. Find the fourth, fifth, and sixth terms of the sequence:

3, 4, 7, ____, ____, ____, …

We find each term by adding the two preceding terms. Three and 4 were added to get the third term, 7. Now we add 4 and 7 to find the fourth term, 11.

3, 4, 7, __11__, ____, ____, …

We continue adding the two preceding terms. The sum of 7 and 11 is 18. The sum of 11 and 18 is 29.

3, 4, 7, __11__, __18__, __29__, …

18. A famous sequence in mathematics is the **Fibonacci sequence,** which follows a pattern similar to the sequence in Example 5. Many patterns found in nature fit the Fibonacci sequence. Below we show the first six terms of the Fibonacci sequence. Find the next three terms.

1, 1, 2, 3, 5, 8, ____, ____, ____, …

We have studied patterns in sequences. There are also patterns between pairs of numbers. A **function table** shows the relationship (or function) between related pairs of numbers. Below we show a function table that shows the relationship between gallons and quarts. A function table can be written vertically or horizontally, as shown.

Gallons	Quarts
1	4
2	8
3	12
5	20

Gallons	1	2	3	5
Quarts	4	8	12	20

Predict Eight gallons of milk is how many quarts of milk?

Example 6

Yosseline is constructing a brick walkway one section at a time. The table below shows the relationship between the numbers of sections and the numbers of bricks she used.

Number of Sections	Number of Bricks
3	36
5	60
8	96

Describe the relationship between the number of sections and the number of bricks. If Yosseline wants to construct a 50-section walkway, how many bricks does she need in all?

By dividing the number of bricks by the number of sections, we find that there are **12 bricks per section** of walkway. A 50-section walkway would require 50 × 12 bricks, or **600 bricks** in all.

Refer to this function table to answer problems **19** and **20:**

In	6	8	10
Out	2	4	6

19. Multiple Choice What does the function do to each "in" number?

   A It divides by 3.   **B** It adds 4.

   C It subtracts 4.   **D** It adds 2.

20. If 15 is used as an "in" number, what will be the "out" number?

Refer to this function table to answer problems **21** and **22:**

Hours Worked	Dollars Paid
5	$40
8	$64
10	$80

21. Describe how to find dollars paid if you know the number of hours worked.

22. If the table shows D'Marcus's pay rate, how much money would he earn in a week if he worked 25 hours?

23. Emelda jogs three miles a day. The table below shows how many miles she would jog in a given number of days. Copy the table on your paper and write in the missing numbers. Then use a calculator to predict the number of miles she would run in a year.

Number of Days	Miles Jogged
2	6
3	
5	
8	