

# Multiplying by Two-Digit Numbers



#### facts

#### Power Up G

#### count aloud

Count by 12s from 12 to 120.

### mental math

- **a. Measurement:** From shoulder to fingertips, Autumn's arm was 2 feet 2 inches long. How many inches is this?
- **b.** Geometry: A hexagon has six sides. If each side of a hexagon is 34 millimeters long, what is the distance around the hexagon?
- **c. Fractional Parts:** Wendy has traveled in  $\frac{6}{10}$  of the 50 states. How many states is this?
- **d. Number Sense:**  $3\frac{1}{4} 1\frac{1}{4}$
- e. Fractional Parts: How many years is  $\frac{1}{4}$  of a century?
- **f. Percent:** 25% of 24
- g. Percent: 10% of 20
- **h. Calculation:**  $\frac{1}{2}$  of 20, + 2,  $\div$  2, + 2,  $\div$  2, + 2,  $\div$  2

# problem solving

In the first 6 games of the season, the Rio Vista football team won 4 games and lost 2 games. They won their seventh game by 8 points. Altogether, the team played 10 games during the season. What is the greatest number of games the Rio Vista team could have won during the season? Is it certain that the Rio Vista team won more than half their games during the season?

## Focus Strategy: Use Logical Reasoning

**Understand** We are told the football team played 10 games during the season. In the first 6 games, they won 4 and lost 2. The team won the next game (the seventh game) by 8 points. We are asked to find how many games the team could have won and whether it is certain the team won more than half their games.

In this problem we are given *irrelevant information*, that is, information that does not help us solve the problem. We ignore irrelevant information when carrying out our solution. While we need to know that the football team won the seventh game, the number of points that the team won by is irrelevant.

**Plan** We use logical reasoning to solve the problem.

**Solve** The team won 4 of the first 6 games, and we know they won the seventh game. So the team won 5 games out of its first 7. This leaves 3 more games that the team could have won. If we assume the team won all 3 of those games, they would have 5 + 3 = 8 wins, which is the most wins they could have during the season.

To find whether it is certain the team won more than half their games, we must assume that the team lost their last 3 games. This would give the team a record of 5 wins and 5 losses. Five wins is exactly half of the games in the season. Thus, it is not certain the team won more than half their games.

**Check** We found the team could win 8 games at most during the season, but it is not certain they would win more than half their games. We know our answers are reasonable because the team could win all 3 of their final games, or they could lose all 3 games. We first found the team's record assuming they won their final 3 games. Then we found the team's record assuming they lost their final 3 games.

# **New Concept**

When we multiply by a two-digit number, we really multiply twice. We multiply by the tens, and we multiply by the ones. Here we multiply 43 by 12. Since 12 is 10 + 2, we may multiply 43 by 10 and 43 by 2. Then we add the products.

$$\begin{array}{c}
43 \\
\times 12
\end{array} \text{ is the same as } \begin{array}{c}
43 \\
\times 10 \\
\hline
430
\end{array} \begin{array}{c}
\times 2 \\
\hline
430
\end{array} \begin{array}{c}
\times 2 \\
\hline
430
\end{array} \begin{array}{c}
\times 3 \\
\times 6 \\
\hline
430 + 86 = 516
\end{array}$$

When we multiply by a two-digit number, we do not need to separate the problem into two problems before we start.

#### Example '

Multiply: 43 × 12

First we multiply 43 by the 2 of 12. We get 86 and we write the 86 so the 6 is in the ones column under the 2.

$$\begin{array}{r}
43 \\
\times 12 \\
\hline
86
\end{array}$$

Next, we multiply 43 by the 10 of 12. We get 430, which we may write below the 86. Then we add 86 to 430 and find that  $43 \times 12$  equals **516.** The numbers 86 and 430 are called **partial products.** The number 516 is the final product. Below are two ways we may show our work:

Some people do not write the trailing zero in the second partial product. In the method on the right, the 0 of 430 is omitted from the second partial product. We begin writing the partial product one place to the left. The 43 means "43 tens."

## Reading Math

Use the steps below to multiply by a two-digit number:

- **1.** Multiply by the ones.
- **2.** Multiply by the tens.
- **3.** Add the partial products.

## Example 2

A restaurant chain purchased 95 pounds of potatoes for each of its 26 locations. About how many pounds of potatoes were purchased altogether?

We are not asked for an exact number, so we can estimate. If we round 95 pounds up to 100 pounds and round 26 pounds to 30, then we estimate that the total number of pounds of potatoes is 3000 pounds.

Analyze Write the estimated amount of potatoes as a fractional part of a ton. (*Hint:* 2000 pounds equals 1 ton.)

#### **Example 3**



SaxonMath.com/ Int5Activities for a calculator activity.

#### At \$0.35 each, what is the cost of two dozen pencils?

We multiply \$0.35 by 24. We ignore the dollar sign and the decimal point until we have a final product.

After multiplying, we place the decimal point. Since we multiplied cents, we show cents in the final product by placing the decimal point so that there are two digits to the right of the decimal point. The cost is **\$8.40**.

The multiplication algorithm presented in this lesson is based on the **Distributive Property.** The Distributive Property applies to situations in which a sum is multiplied, such as

$$25 \times (10 + 2)$$

According to the Distributive Property, we have two choices when multiplying a sum:

Choice 1: Find the sum; then multiply.

Choice 2: Multiply each addend; then add the products.

Here we illustrate these choices:

$$25 \times (10 + 2)$$
  
 $25 \times 12$  or  $(25 \times 10) + (25 \times 2)$ 

Both choices result in the same answer (which in this case is 300).

### Example 4

Benito wants to multiply 35 by (20 + 4). Using the Distributive Property, show his two choices. Then find each answer.

Here are Benito's two choices:

$$35 \times (20 + 4)$$
  
35 × 24 or  $(35 \times 20) + (35 \times 4)$ 

Now we find each answer:

Notice that 700 and 140 appear as partial products in both methods.

## Lesson Practice

Multiply:

- **a.** 32 **b.** \$0.62 **c.** 48  $\times$  12  $\times$  23  $\times$  64
- **d.** 246 **e.** \$1.47 **f.** 87  $\times$  22  $\times$  34  $\times$  63
- **g.** Musoke wants to multiply 12 by (20 + 3). Show her two choices for multiplying. Find each answer.
- h. Estimate Early one morning, a bakery shipped 11 boxes of bagels to local supermarkets. Each box contained 24 bagels. Show two different ways to estimate the number of bagels that were shipped that morning. Then choose one of the ways and explain why it represents a better estimate.

# Written Practice

Distributed and Integrated

**1.** The numbers of visitors to the school science fair are shown in the table:

#### **Science Fair**

Day	Number of Visitors
Wednesday	47
Thursday	76
Friday	68
Saturday	

The total attendance for the four days was 320 visitors. How many visitors attended the science fair on Saturday?

\*2. To mail the letter, Yai-Jun used one 39-cent stamp and three 23-cent stamps. How many cents did it cost to mail the letter?

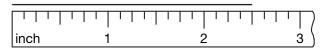
\*3. Represent Draw a diagram to illustrate and solve this problem:

Arthur ate  $\frac{3}{4}$  of the 60 raisins. How many raisins did he eat?

What percent of the raisins did he eat?

- **4.** Represent Write  $(1 \times 1000) + (1 \times 1)$  in standard form.
- **5.** Represent Use words to name 1760.
- \*6. Represent Draw a circle. Shade all but one sixth of it. What percent of the circle is *not* shaded?
  - **7. Represent** Use digits to write sixty-two thousand, four hundred ninety.
- \*8. Multiple Choice The perimeter of the Khafne Pyramid in Egypt is 2835 feet. When we count by hundreds, we find that 2835 is closest to which number?
  - **A** 2000
- **B** 2700
- **C** 2800
- **D** 2900

9. How long is the line segment below?



\* 10. Analyze Below are two stacks of coins. If some coins were taken from the taller stack and added to the shorter stack until the stacks were even, how many coins would be in each stack?





**11.** Compare: 
$$\frac{1}{2}$$
 of 10  $\bigcirc$   $\frac{1}{3}$  of 12

**12.** 
$$(1+2+3+4+5) \div 5$$

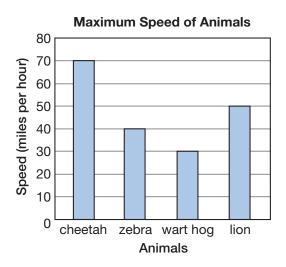
$$20. \frac{\$6.00}{8}$$

**22.** 
$$9x = 4275$$

**23.** 
$$3 + \frac{1}{4} + 2\frac{2}{4}$$

**24.** 
$$\left(5\frac{5}{8} - 3\frac{3}{8}\right) - 1\frac{1}{8}$$

- **25.** In the running long jump, S'Mira jumped 16 feet 9 inches. How many inches did she jump? (One foot equals 12 inches.)
- **26.** Ajani needs to multiply 15 by (20 + 4). Using the Distributive Property, show his two choices and the final product.
- **27.** This table shows how fast some animals can run:
  - **a.** Which two speeds are used to find the range of the data?
  - **b.** What is the median speed of the animals?
  - c. Which animal has a maximum speed that is closest to the average speed of all of the animals shown in the graph?



- \*28. Represent Write 205,000 in expanded notation.
- **\*29.** Estimate The math book was  $11\frac{1}{4}$  inches long. Round  $11\frac{1}{4}$  inches to the nearest inch.
- 30. The distance between Kenley's and Bernardo's house is 24 miles. Last month, Kenley drove from his house to Bernardo's house, and back again, 9 different times. What is a reasonable estimate of the number of miles Kenley drove? Explain your answer.



# Naming Numbers Through Hundred Billions



#### facts

## Power Up F

#### count aloud

Count by 6s from 6 to 60. Count by 60s from 60 to 300.

### mental math

**a. Time:** 2 minutes 10 seconds is how many seconds?

**b. Measurement:** The window was 4 feet 2 inches from top to bottom. What is this length in inches?

**c. Measurement:** There are 16 ounces in a pound. How many ounces are in 3 pounds?

**d. Number Sense:**  $1\frac{1}{8} + \frac{7}{8}$ 

e. Time: 50% of a minute

f. Time: 25% of a minute

g. Time: 10% of a minute

**h. Calculation:**  $6 \times 6$ , -6,  $\div 6$ , +5,  $\div 5$ ,  $\times 7$ , +1,  $\div 3$ 

# problem solving

Alicia and Barbara attended the carnival together. Alicia paid the admission prices, which were \$8 per person. Barbara paid for the rides and the snacks, which were \$20 altogether.

After the carnival, Alicia and Barbara decided to share the costs equally. Which girl paid more than her share at the carnival? Which girl paid less than her share at the carnival? How could they settle the difference so that they each pay an equal amount?

## Focus Strategies: Make a Model; Act It Out

**Understand** We are told that Alicia and Barbara each paid for items at a carnival. We are asked to find which girl paid more than her share and which paid less than her share. We are also asked to find how the girls could settle the difference so that they each would pay an equal amount.

Plan We can act out the situation by using our money manipulatives to model the problem. Let's suppose Alicia and Barbara each start with \$20. If the girls start with the same amount and then share costs equally, they should have equal amounts of money left over after paying for items at the carnival.

Solve Alicia paid the admission prices, which were \$8 per person, or \$16 altogether. We take away \$16 from Alicia's money. This leaves Alicia with \$4. Barbara paid for snacks and rides, which cost \$20 altogether. We take away \$20 from Barbara's money, which leaves her with no money. We see that if Alicia gives Barbara \$2 from the \$4 she has remaining, each girl would have \$2, and they would be "even."

If we add up the prices, we see that the girls spent \$36 altogether. Half of that amount is \$18. This means that before settling the difference, Alicia paid \$2 less than her share, and Barbara paid \$2 more than her share.

**Check** We know that our answers are reasonable because the girls spent \$36 altogether, which means each girl should have spent \$18. However, Alicia paid \$16 and Barbara paid \$20. So Alicia spent \$2 less than she should have, and Barbara spent \$2 more than she should have.

In our solution, we assumed that each girl started with the same amount of money. If the girls started with different amounts, would they still equally share the cost after paying for the items at the carnival? Explain your answer.

# New Concept

The diagram below shows the values of the first twelve whole-number places:



an online activity.

hundred billions	hundred millions	hundred thousands	hundreds
ten billions	ten millions	ten thousands	tens
billions	millions	thousands	ones
,	;	;	

**Discuss** Describe how the millions place and the thousands place compare.

Drawing the place-value diagram a different way emphasizes the repeating pattern of place values.

В	illior	ns		Ν	1illior	าร		Tho	ousa	nds	a	Unit	s (O	nes)
hundreds	tens	seuo	billions comma	hundreds	tens	ones	millions comma	hundreds	tens	ones	thousands comm	hundreds	tens	seuo

Analyze How many millions are equal to one billion?

We see that the pattern of **ones, tens, hundreds** repeats through the thousands, millions, and billions.

#### Example 1

Which digit shows the number of hundred billions in 987,654,321,000?

Moving from right to left, the pattern of ones, tens, hundreds continues through the thousands, millions, and billions. The digit in the hundred-billions place is **9.** 

## Example 2

What is the value of the 2 in the number 12,345,678?

A 2,000,000 B 2000

C 2

D 20,000

The value of a digit depends upon its place in the number. Here the 2 means "two million." The correct choice is **2,000,000.** 

To name whole numbers with many digits, it is helpful to use commas. To insert commas, we count digits from the right-hand side of the whole number and put a comma after every three digits.

We write a comma after the millions place and after the thousands place. When reading a number with two commas, we say "million" when we come to the first comma and "thousand" when we come to the second comma.

#### **Reading Math**

Newspapers and magazines usually use the short word form of very large numbers, such as:

mbers, such a 105 million

260 billion

3 trillion

Using words, we name this number as follows:

eighty-seven million, six hundred fifty-four thousand, three hundred twenty-one

#### Example 3

Use words to name 1345200.

We first put the commas in the number: 1,345,200. Then we name the number as one million, three hundred forty-five thousand, two hundred.

#### Example 4

Use digits to write one hundred thirty-four billion, six hundred fifty-two million, seven hundred thousand.

We write the number as 134,652,700,000.

## **Example 5**

Write 2,500,000 in expanded notation.

We write 2 times its place value plus 5 times its place value.

$$(2 \times 1,000,000) + (5 \times 100,000)$$

**Verify** Two million, five hundred thousand can be written as 2.5 million. Explain why.

## Lesson Practice

In problems a-d, name the value of the place held by the zero in each number.

**a.** 345,052

**b.** 20,315,682

**c.** 1,057,628

**d.** 405,176,284

e. In 675,283,419,000, which digit is in the ten-billions place?

f. Multiple Choice In which of the following numbers does the 7 have a value of seventy thousand?

**A** 370,123,429

**B** 1,372,486

**C** 4,703,241

**D** 7,000,469

g. Use words to write the value of the 1 in 321,987,654.

**Represent** Use words to name each number:

**h.** 21462300

i. 19650000000

**Represent** Use digits to write each number:

- j. nineteen million, two hundred twenty-five thousand, five hundred
- k. seven hundred fifty billion, three hundred million
- **I.** two hundred six million, seven hundred twelve thousand, nine hundred thirty-four
- m. Represent Write 7,500,000 in expanded notation.

# **Written Practice**

Distributed and Integrated

- **1.** Thao made 5 dozen baked apples and gave 24 to a friend. How many baked apples did she have left?
- **2.** Marco weighs 120 pounds. His younger brother weighs one half as much. How much does his brother weigh?
- **3.** Hope bought a chain for \$3.60 and a lock for \$4. How much should she get back in change from a \$10 bill?
- **4.** In 1607, Captain John Smith led a group of British explorers who settled in Jamestown, Virginia. How many centuries are there from the year 1607 to the year 2007?
  - 5. Represent Write (1  $\times$  100) + (4  $\times$  10) + (8  $\times$  1) in standard form.
- \*6. Represent Draw a rectangle that is 2 inches long and 1 inch wide.

  Shade all but three eighths of it. What percent of the rectangle is not shaded?
  - **7. Represent** Use words to name the number 250,000.
- \*8. This picture shows three stacks of books. If the stacks were made equal, how many books would be in each stack? Explain your answer.



\*9. Which digit in 789,456,321 shows the number of hundred millions?

- **10.** Round 1236 to the nearest hundred.
- **11.** Name the value of the place held by the zero in 102,345,678.  $^{(52)}$

19. 
$$\frac{4328}{4}$$

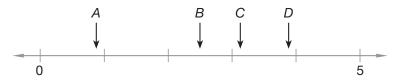
**20.** 
$$\frac{5670}{10}$$

**21.** 
$$\frac{$78.40}{4}$$

**22.** 
$$\frac{3}{10} + 2 + 1 \frac{4}{10}$$

**23.** 
$$5\frac{3}{4} - \left(2\frac{3}{4} - 2\right)$$

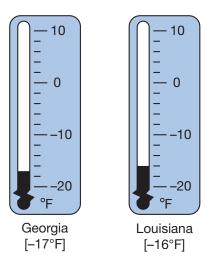
\*25. Connect Which arrow could be pointing to  $3\frac{9}{10}$  on the number line below?



- **26.** Tuan needed to multiply 25 by 24. He thought of 24 as 20 + 4. Using the Distributive Property, show two choices Tuan has for multiplying the numbers.
- \*27. Maura counted the number of trees on each property on her block. The results are listed below.
  - a. Make a line plot to display these data:

- **b.** Name the median, mode, and range.
- c. Name two outliers.
- d. Name a data cluster.

- \*28. Represent Write three million, two hundred thousand in expanded notation.
  - **29.** The thermometers show the lowest temperatures ever recorded in two states.



The two temperatures differ by what number of degrees?

\*30. Cameron's age in years is 2 fewer years than 10 times his brother's age. Cameron's brother is 1 year old. How old is Cameron?



Saturn is about 1352550000 kilometers away from the sun.

- a. Rewrite the number and insert commas.
- **b.** Which digit is in the hundred-millions place?
- **c.** Underline the digit in the ten-thousands place.
- d. Use words to write the number.



- Perimeter
- Measures of a Circle



#### facts

# count aloud

### mental math

Power Up F

Count by 6s from 6 to 60. Count by 60s from 60 to 360.

- **a. Time:** The movie was 2 hours 15 minutes long. How many minutes is that?
- **b. Money:** Vikas earned \$15.00 for raking leaves. He spent \$4.75 of his earnings on a comic book. How much money is left over?
- **c. Measurement:** 1000 meters is one kilometer. How many meters is 25% of a kilometer?
- **d. Number Sense:**  $2\frac{1}{2} + 2\frac{1}{2}$
- **e. Time:** How many minutes is  $1\frac{1}{2}$  hours? ...  $2\frac{1}{2}$  hours?
- **f. Percent:** The sale price of the tennis racket is 50% of \$41. What is the sale price?
- **g. Measurement:** The bicycle is 5 feet 4 inches long. How many inches is that?
- **h. Calculation:**  $\frac{1}{2}$  of 100,  $\div$  2,  $\div$  5,  $\div$  5,  $\times$  10,  $\div$  5

# problem solving

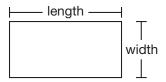
Choose an appropriate problem-solving strategy to solve this problem. Alvin finds that he can arrange objects into triangular patterns of 3, 6, and 10 objects, respectively. Alvin finds that he can also arrange objects into square patterns of 4, 9, 16, and 25 objects, respectively. Find the smallest number of objects greater than 1 that Alvin can arrange into either a triangular pattern or a square pattern.

# New Concepts

#### **Perimeter**

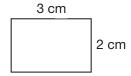
When line segments enclose an area, a polygon is formed. We can find the distance around a polygon by adding the lengths of all the segments that form the polygon. The distance around a polygon is called the **perimeter**.

We should note that the word *length* has more than one meaning. We have used length to mean the measure of a segment. But length may also mean the longer dimension of a rectangle. We use the word *width* to mean the shorter dimension of a rectangle.



#### Example 1

What is the perimeter of this rectangle?



The perimeter is the distance around the rectangle. This rectangle has a length of 3 cm and a width of 2 cm. The four sides measure 2 cm, 3 cm, 2 cm, and 3 cm.

$$2 \text{ cm} + 3 \text{ cm} + 2 \text{ cm} + 3 \text{ cm} = 10 \text{ cm}$$

We added the lengths of the sides and found that the perimeter is **10 cm.** 

Notice that to find the perimeter, we added the length plus the width plus the length plus the width. In other words, we added two lengths plus two widths. Using *I* for length, *w* for width, and *P* for perimeter, we can express the formula for the perimeter of a rectangle this way:

$$P = 2I + 2w$$

## Example 2

Use the formula on the next page to find the perimeter of the rectangle in Example 1.

$$P = 2I + 2w$$
  
 $P = (2 \times 3) + (2 \times 2)$   
 $P = 10 \text{ cm}$ 

A **regular polygon** has sides equal in length and angles equal in measure. For example, a square is a regular quadrilateral. Below we show some regular polygons:



If we know the length of one side of a regular polygon, we can find the perimeter of the polygon by multiplying the length of one side by the number of sides.

**Generalize** What formula could be used to find the perimeter of any regular polygon?

### **Example 3**

## What is the perimeter of this regular triangle?



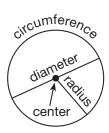
The perimeter is the total of the lengths of the three sides. We can find this by multiplying the length of one side of the regular triangle by 3.

$$P = 3 \times \text{side length}$$
  
  $3 \times 12 \text{ inches} = 36 \text{ inches}$ 

Analyze What is the perimeter of the triangle in yards?

# Measures of a Circle

A **circle** is a smooth curve. The length of the curve is its **circumference**. So the circumference of a circle is the perimeter of the circle. The **center** of the circle is the "middle point" of the area enclosed by the circle. The **radius** is the distance from the center to the curve. The **diameter** is the distance across the circle through its center. Thus, the diameter of a circle is twice the radius.





## **Measuring Circles**

Materials needed:

- Lesson Activity 34
- various circular objects such as paper plates, cups, wheels, and plastic kitchenware lids
- ruler, cloth tape measure, string, or masking tape

Make a list of circular objects at school and home. Measure the diameter, radius, and circumference of each object. Record the results in the table on **Lesson Activity 34.** 

## Lesson Practice

- a. What is the length of this rectangle?
- **b.** What is the width of the rectangle?
- c. What is the perimeter of the rectangle?
- **d.** What is the perimeter of this right triangle?
- 5 cm 3 cm

3 in.

5 in.

**e. Generalize** Use a formula to find the perimeter of this square:



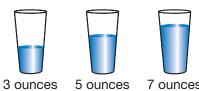
- **f.** What do we call the perimeter of a circle? Do we use units, square units, or cubic units to measure this perimeter?
- **g.** What do we call the distance across a circle through its middle?
- **h.** If the radius of a circle is 6 inches, what is the diameter of the circle?

# Written Practice

Distributed and Integrated

**1.** A baker used fifteen of three dozen eggs to make six spice cakes and five loaves of sourdough bread. How many eggs were not used?

- \*2. Analyze There are 13 players on one team and 9 players on the other team. If some of the players from one team join the other team so that the same number of players are on each team, how many players will be on each team? Explain your reasoning.
- 3. Represent If  $\frac{1}{3}$  of the 30 students walked home, how many students walked home? What percent is this? Draw a diagram to illustrate and solve this problem.
- \*4. Analyze If water is poured from glass to glass until the amount of water in each glass is the same, how many ounces of water will be in each glass? Explain your reasoning.



- **5. Multiple Choice** In the number 123,456,789,000, the 2 means which of the following?
  - A 2 billion
- **B** 20 billion
- C 200 billion
- **D** 2000 billion
- \*6. List Which factors of 8 are also factors of 12?
- **7.** How many decades were between the years 1820 to 1890?  $_{(28,35)}$ 
  - **8.** Represent Use digits to write nineteen million, four hundred ninety thousand.

**9.** 6 + 
$$\left(4\frac{2}{3}-2\right)$$

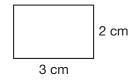
**10.** 
$$4\frac{2}{3} - \left(2\frac{2}{3} + 2\right)$$

**13.** 
$$5t = 500$$

343

- \*23. Multiple Choice When we count by tens, we find that 1236 is closest to which number?
  - **A** 1230
- **B** 1240
- **C** 1200
- **D** 1300

- **24.** What is the length of this rectangle?
- \*25. Generalize Use a formula to find the perimeter of this rectangle.



- **26.** To multiply 35 by 21, Nancee thought of 21 as 20 + 1. Show two choices Nancee has for multiplying the numbers.
- **27. Represent** Write 2,050,000 in expanded notation.
- **28.** Represent Draw an equilateral triangle.
- **29.** Alba found the circumference of the soup can to be  $8\frac{5}{8}$  inches. Round  $8\frac{5}{8}$  inches to the nearest inch.
- \*30. The highest elevation above sea level in each of four states is shown in the pictograph. The elevations have been rounded to the nearest hundred feet.

State	Highest Elevation (in feet)
Mississippi	
New Jersey	
Illinois	
Michigan	

Key: = 200 feet

- a. Analyze Which state has a highest elevation of about 2000 feet?
- **b.** Write numbers to represent the elevations and order the numbers from greatest to least.
- c. Which elevation is nearest sea level?



# Dividing by Multiples of 10

# Power Up

#### facts

# mental math

Power Up G

**a. Money:** \$1.00 – \$0.33

**b. Number Sense:**  $7\frac{1}{2} + 1\frac{1}{2}$ 

c. Money: What coin equals 50% of 50¢?

d. Money: What coin equals 10% of 50¢?

e. Measurement: 4 feet 2 inches is how many inches?

**f. Geometry:** Each side of the square is  $1\frac{1}{2}$  inches long. What is the perimeter of the square?

**g. Measurement:** The temperature reached a high of 82°F. Then it dropped to a low of 68°F. What was the difference between the high and low?

**h. Calculation:**  $6 \times 6$ , -1,  $\div 5$ ,  $\times 2$ , +1,  $\div 3$ ,  $\times 2$ 

# problem solving

Choose an appropriate problem-solving strategy to solve this problem. Carlos planted thirty-six carrots in his garden. He arranged the carrots into a square array of rows and columns. How many carrots are in each row?

# **New Concept**

#### **Reading Math**

We use four steps for long division:

- 1. Divide
- 2. Multiply
- 3. Subtract
- 4. Bring down

In this lesson we will begin to divide by two-digit numbers that are multiples of 10. Multiples of 10 are the numbers 10, 20, 30, 40, 50, and so on. In later lessons we will practice dividing by other two-digit numbers.

We will continue to follow the four steps of the division algorithm: divide, multiply, subtract, and bring down. The divide step is more difficult when dividing by two-digit numbers because we may not quickly recall two-digit multiplication facts. To help us divide by a two-digit number, we may think of dividing by the first digit only.

To help us divide this:  $30\overline{)75}$ 

... we may think this: 3)7

We use the answer to the easier division  $\frac{2}{30}$  R 15 for the answer to the more difficult  $\frac{30}{75}$  division. Since  $\frac{3}{7}$  is 2, we use 2 in the division answer. We complete the division by doing the multiplication and subtraction steps.

Notice where we placed the 2 above the box. Since we are dividing 75 by 30, we place the 2 above the 5 of 75 and not above the 7.

The 2 above the 5 means there are two 30)75 30s in 75. This is the correct place.

It is important to place the digits in the quotient properly.

### Example 1

The staff arranged 454 chairs in the school gymnasium. Each row of the arrangement contained 30 chairs, except the last row. How many complete rows of chairs are in the arrangement? How many chairs are in the last row?

#### **Thinking Skill**

#### Connect

Why do we write a 1 above the tens place in the quotient? We follow the four steps: divide, multiply,  $\frac{15}{30}$  R 4 subtract, and bring down. We begin by  $\frac{30}{454}$  finding  $\frac{30}{45}$ . If we are unsure of the answer, we may think "3)4" to help us  $\frac{30}{154}$  with the division step. We divide and write  $\frac{150}{4}$  "1" above the 5 of 454. Then we multiply,

subtract, and bring down. Since we

brought down a digit, we divide again. This time we divide 154 by 30. To help us divide, we may mentally remove the last digit from each number and think "3)15." We write "5" above the box, and then multiply and subtract. The answer to the division is 15 R 4. This quotient means there are **15 rows** of 30 chairs and one row of **4 chairs**.

Recall that we check a division answer	15
by multiplying the quotient by the divisor	× 30
and then adding any remainder. The	450
result should equal the dividend.	+ 4
	454

#### **Example 2**

Mr. Gibson has a small grove of 18 young orange trees that produced 782 pounds of oranges this year. Estimate the average number of pounds of oranges produced by each tree.

Round 18 trees up to 20 and 782 pounds up to 800 pounds and divide. We find that on average, each tree produced **about 40 pounds** of oranges.

### Example 3

Taryn bought 20 bread rolls for \$4.60. What was the cost for each roll?

When dividing money by a whole number,	\$ .23
we place the decimal point in the quotient	20)\$4.60
directly above the decimal point in the	4 0
dividend. Then we ignore the decimal	<del>6</del> 0
points and divide just as we would divide	6 0
whole numbers. By adding a zero before	0
the decimal point, we get an answer of	
\$0.23 for each roll.	

**Justify** Explain why the answer is reasonable.

## Lesson Practice

Divide:

- **a.** 30)\$4.20
- **b.** 60)725
- **c.** 40)\$4.80

- **d.** 20)\$3.20
- **e.** 50)610
- **f.** 10)345
- **g.** Show how to check this division answer. Is the answer correct?
- 23 R 5 40)925
- **h.** Quan bought 18 eggs at the supermarket for \$4.60. Estimate the cost per egg. Show how you found your answer.

# Written Practice

Distributed and Integrated

- 1. Camilla went to the store with \$5.25. She bought a box of cereal for \$3.18 and a half gallon of milk for \$1.02. How much money did Camilla have left? Explain why your answer is reasonable.
- 2. Round 1236 to the nearest ten.

- **\*3.** Represent A yard is 36 inches. How many inches is  $\frac{2}{3}$  of a yard? Draw a diagram to illustrate the problem.
- **\*4. Multiple Choice** The 7 in 987,654,321 means which of the following?
  - **A** 700
- **B** 7,000,000
- **C** 700,000
- **D** 7000
- **5.** Represent Draw two circles. Shade  $\frac{1}{2}$  of one and  $\frac{2}{4}$  of the other. What percent of a circle is  $\frac{2}{4}$  of a circle?
- **6. a.** How many cents is  $\frac{1}{4}$  of a dollar?
  - **b.** How many cents is  $\frac{2}{4}$  of a dollar?
  - \*7. Represent Use words to name the number 3,150,000,000.
  - **8.** List Which factors of 9 are also factors of 12?

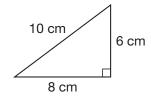
**15.** 
$$25\frac{1}{4} + 8\frac{2}{4}$$

**16.** 
$$36\frac{2}{3} - 17\frac{2}{3}$$

- **21.** Thirty-six children were seated at tables with four children at each table. How many tables with children were there?
- **22.** If the diameter of this circle is 30 millimeters, then what is the radius of the circle?



**23.** What is the perimeter of this right triangle?



\*24. Use a ruler to find the length of this rectangle in inches:



- **25.** What year was five decades after 1896, the year the first modern olympics were held in Athens, Greece?
  - **26.** Analyze Irina wants to multiply 150 by 12. She thinks of 12 as 10 + 2. Using the Distributive Property, show two ways Irina can multiply the numbers. What is the product?
- **27.** Here is a sequence of numbers we say when counting by sixes:  $\binom{1}{42}$

Here is the same sequence in a function table:

Position of Term	1	2	3	4	5
Term	6	12	18	24	30

- **a.** Write a rule that describes how to find a term if you know its position.
- **b.** What number is the twentieth term of the sequence?
- **28.** Sergio earns \$14 an hour for working up to 8 hours a day, and \$21 an hour for every hour he works beyond 8 hours. How much does Sergio earn for a day he works 11 hours?
- \*29. Conclude Could a triangle with sides 8 cm, 6 cm, and 8 cm long be a scalene triangle? Why or why not?
- **30.** What is a reasonable estimate for the quotient of  $776 \div 38$ ? Explain your answer.



# Multiplying by Three-Digit Numbers



#### facts

# mental math

## Power Up F

- **a. Time:** Soccer practice started at 4:15 p.m. It ended 1 hour 20 minutes later. What time did soccer practice end?
- **b. Percent:** How many hours is 50% of a day?
- c. Percent: How many hours is 25% of a day?
- d. Measurement: Five feet six inches is how many inches?
- **e. Geometry:** Each side of the triangle is  $3\frac{1}{3}$  inches long. What is the perimeter of the triangle?
- **f. Percent:** Mason deposited 25% of \$40 into a savings account. How much is 25% of \$40?
- **g. Number Sense:**  $7\frac{1}{8} + 1\frac{7}{8}$
- **h. Calculation:**  $6 \times 8$ , + 1,  $\div 7$ , + 2,  $\div 3$ , + 1,  $\div 2$

# problem solving

Choose an appropriate problem-solving strategy to solve this problem. Anthony has \$19 to spend at the school's book fair. Fiction books are \$3 each, science books are \$4 each, and art books are \$5 each. How many of each kind of book can he buy with \$19? What are the combinations of books that would cost exactly \$12? If Anthony buys four times as many fiction books as science books, how much money will he spend in all?

# **New Concept**

When we multiply by a three-digit number, we actually multiply three times; we multiply by the hundreds, we multiply by the tens, and we multiply by the ones. We demonstrate this on the next page with the multiplications for finding  $234 \times 123$ .

#### **Thinking Skill**

#### Generalize

Would we use the same multiplication algorithm if we were multiplying by ten digits? Why or why not?

$$23,400 + 4680 + 702 = 28,782$$

We do not need to separate a three-digit multiplication problem into three problems before we start. We may do all the multiplication within the same problem.

### **Example 1**

Multiply: 234 × 123



 $4680 \leftarrow$  Then we multiply by the 20 of 123. The zeros need not  $23400 \leftarrow$  Then we multiply by the 100 of 123. be written.

28782 ← We add the three partial products to find the total product.

We should know how to perform pencil-and-paper computations with many digits. However, most people would use a calculator to do arithmetic that would be time consuming to do by hand.

**Explain** Describe or demonstrate how we could perform the multiplication with a calculator.

## Example 2

A restaurant served 356 glasses of juice during brunch. The capacity of each glass was 250 milliliters. About how many milliliters of juice did the restaurant serve during brunch?

The word "about" in the question means we can estimate. To estimate a product, we may get closer to the exact product by rounding one factor up and the other factor down. We round 250 milliliters up to 300 milliliters and 346 glasses down to 300 glasses.

$$300 \times 300 = 90,000$$

The restaurant served about **90,000 mL** of juice.

**Analyze** About how many liters of juice did the restaurant serve? Explain how you know. (*Hint:* 1000 milliliters = 1 liter).

## Lesson Practice

Find each product:

351

**d.** Use compatible numbers to find the product.

 $705 \times 678$ 

**e. Estimate** What is a reasonable estimate for the quotient of 739 ÷ 18? Explain your answer.

# Written Practice

Distributed and Integrated

- 1. Cruz bought a fruit plate for \$4.65 and a drink for \$1.90. He paid for the food with a \$10 bill. How much should he get back in change? Explain why your answer is reasonable.
- 2. Represent Draw a diagram to illustrate and solve this problem:

  There are 276 pages in the book. If Navarro has read three fourths of the book, how many pages has he read?
- **3.** The Loire River in Europe is 26 miles shorter than the Ubangi River in Africa. The Loire River is 634 miles long. Find the length of the Ubangi River by writing and solving an equation.
- \*4. Which digit in 98,765,432 is in the ten-millions place?
- **5.** Amanda can jump across a rug that is 2 yards 3 inches long. How many inches is 2 yards 3 inches? (A yard is 36 inches.)
- \*6. Represent Draw a circle and shade all but one third of it. What percent of the circle is shaded?
- \*7. Represent Use digits to write six hundred seventy-nine million, five hundred forty-two thousand, five hundred.
- **8.** 60)\$7.20

**9.** 70) 850

**10.** 80) 980

**11.** 234 × 123

**12.** \$3.75

- 13. 604 (55) × 789
- \*14. Each side of this square is 10 mm long. Use a formula to find the perimeter of the square.

10 mm

Use mental math to answer problems 15-20.

**15.** 
$$400 \times 800$$

**16.** 
$$60 \times 500$$

**17.** 
$$900 \times 90$$

**20.** 
$$\frac{400}{20}$$

**21.** 
$$6\frac{5}{11} + 5\frac{4}{11}$$

**22.** 
$$3\frac{2}{3} - 3$$

**23.** 
$$7\frac{2}{3} - \left(3\frac{1}{3} - 3\right)$$

Use this information to answer problems 24 and 25:

The Arroyo High School stadium can seat 3000 fans. Two thousand, one hundred fifty ticket-holding fans came to the first game. Arroyo won by a score of 35 to 28. Tickets to watch the game cost \$2 each.

- **24.** Altogether, the fans who came to the first game paid how much money Inv. 5) for tickets?
- **25.** At the second game all but 227 seats were filled with fans. How many fans came to the second game?
- \*26. Represent The crowd lining the parade route was estimated to be 1,200,000. Write this number in expanded notation.
  - **27. Represent** Draw an isosceles triangle.
- **28.** If a dollar's worth of dimes is divided into five equal groups, how many dimes would be in each group?
- **\*29.** Estimate A young gecko is  $5\frac{7}{8}$  inches long. Record the length of the gecko to the nearest inch.
- **30.** Estimate What is a reasonable estimate for the quotient of  $689 \div 19$ ? Explain your answer.

Early Finishers Real-World Connection Several park employees gathered data and found that 673 people entered the park in one day. Based on this data, predict how many people will enter the park in a year if it is open six days a week throughout the year.



# Multiplying by Three-Digit Numbers that Include Zero

# Power Up

#### facts

## mental math

## Power Up G

- **a. Estimation:** Round  $5\frac{7}{8}$  in. to the nearest inch.
- **b. Estimation:** Round  $12\frac{3}{8}$  in. to the nearest inch.
- **c. Estimation:** Round  $9\frac{3}{4}$  in. to the nearest inch.
- **d. Number Sense:** How much is  $600 \div 10? \dots 600 \div 20 \dots 600 \div 30?$
- **e. Time:** How many days is 52 weeks? This number of days is how much less than 1 year?
- **f. Measurement:** One mile is 5280 feet. Dakota jogged the first 4800 feet of the mile, and then she walked the remainder. How far did she walk?
- **g. Percent:** Fifty percent of the 42 children have birthdays in January through June. What is 50% of 42 children?
- **h. Calculation:**  $6 \times 8$ , + 6,  $\div 9$ ,  $\times 7$ , 7,  $\div 5$

# problem solving

Choose an appropriate problem-solving strategy to solve this problem. Tiffany wrote a multiplication problem and then erased some of the digits. She then gave the problem to J'Anna as a problem-solving exercise. Copy Tiffany's multiplication problem and find the missing digits for J'Anna.

# **New Concept**

When we multiply by a three-digit number that has a zero as one of its digits, we may find the product by doing two multiplications instead of three.

#### Example

#### Multiply: $243 \times 120$

When we multiply by a number that ends with a zero, we may write the problem so that the zero "hangs out" to the right.

We place the thousands comma in the final product to get **29,160**.

(Analyze) If we multiply  $243 \times 12$  and then write a zero at the end of the product, will the product be correct? Why or why not?

### Example 2

### Multiply: $243 \times 102$

We may write the two factors in either order. Sometimes one order is easier to multiply than the other. In the solution on the left, we multiplied three times. On the right, we used a shortcut and multiplied only twice. Either way, the product is 24,786.

The shortcut on the right was to "bring down" the zero in the bottom factor rather than multiply by it. If we had not used the shortcut, then we would have written a row of zeros as shown below.

$$\begin{array}{c}
243 \\
\times 102 \\
\hline
486
\end{array}$$
zero in bottom factor
$$\begin{array}{c}
000 \\
\hline
243 \\
\hline
24786
\end{array}$$

In order to use this pencil-and-paper shortcut, we remember to set up multiplication problems so that factors containing zero are at the bottom.

### **Thinking Skill**

#### Verify

What property states that we can multiply two factors in any order?

## **Example 3**

During a minor league baseball game, a vendor sold 120 hot dogs for \$3.25 each. What amount of money did the vendor collect for the sale of those hot dogs?

We ignore the dollar sign and the decimal point until we have finished multiplying. We place the dollar sign and the decimal point in the final product to get \$390.00.

$$3.25$$
 $\times$  120
 $\overline{6500}$ 
 $325$ 
 $\overline{320000}$ 

The vendor sold \$390.00 worth of hot dogs.

**Discuss** Why did we place the decimal point two places from the right?

## Lesson Practice

Multiply:

**d.** 
$$304 \times 120$$

# Written Practice

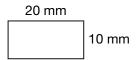
Distributed and Integrated

- **1.** Cantrice and her sister want to buy software for \$30. Cantrice has \$12 and her sister has \$7. How much more money do they need?
- \*2. Represent How many seconds equal three sixths of a minute? Draw a diagram to illustrate and solve the problem.
- 3. Jada's house is 8 blocks from school. How many blocks does she ride her bike to and from school in 5 days? Explain how you found your answer.

- **4.** Analyze When the students got on the buses to go to the picnic, there were 36 on one bus, 29 on another bus, and 73 on the third bus. If students are moved so that the same number are on each bus, how many students will be on each bus?
- \*5. Represent Which digit in 123,456,789 is in the ten-thousands place?
- \*6. Connect The radius of this circle is 5 inches. What is the diameter of the circle?



- \*7. Represent Use digits to write the number three hundred forty-five million, six hundred fourteen thousand, seven hundred eighty-four.
- **8.** Use a formula to find the perimeter of this rectangle:



**10.** 
$$700 \times 400$$

**11.** 
$$234 \times 320$$

**13.** 
$$468 \times 386$$

**14.** 
$$\frac{W}{5} = 6$$

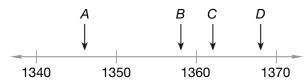
**16.** 2703 
$$\div$$
 9

**17.** 
$$8m = $86.08$$

**21.** 
$$3\frac{5}{6} - 1\frac{5}{6}$$

**22.** 
$$4\frac{1}{8} + 6$$

- **23.** Three weeks and three days is how many days?
- \*24. Connect Which arrow could be pointing to 1362?



- **25.** Use words to name the mixed number  $7\frac{1}{10}$ .
- **26.** Analyze Turi needs to multiply 203 by 150. He thinks of 203 as 200 + 3. Show two ways Turi could multiply these numbers. What is the product?
- \*27. a. Multiple Choice Which of these divisions has no remainder?

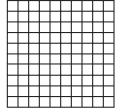
**A** 543 ÷ 9

**B** 543 ÷ 5

**C**  $543 \div 3$ 

**D** 543 ÷ 2

- **b.** Explain how you know.
- **28.** The large square has been divided into 100 small squares.
  - **a.** How many small squares equal  $\frac{1}{4}$  of the large square?
  - **b.** What is  $\frac{1}{4}$  written as a decimal?



- **29.** The circumference of the globe was  $37\frac{3}{4}$  inches. Round the circumference to the nearest inch.
- \*30. Kiersten always uses the same kind of golf ball when she plays golf.

  The golf ball she uses has dimples or small indentions on the surface to help the ball fly farther when hit. The kind of ball Kiersten uses and its relationship to dimples on its surface is shown below.

Number of Golf Balls	1	2	3	4
Number of Dimples	392	784	1176	1568

- a. Generalize Describe the relationship between golf balls and dimples.
- **b. Predict** Kiersten just purchased a box of new golf balls. There are 12 balls in the box. Altogether, how many dimples are on all 12 golf balls in the box?



Section D of the City Park Football Stadium can seat 325 people. The average ticket price for each seat in this section is \$5.50.

- **a.** How much money does the stadium make from this section when all the tickets for one game are sold?
- **b.** If this section were sold out for ten games, how much money would the stadium make from the tickets sold for this section?



# Probability



#### facts

# mental math

### Power Up F

- **a. Estimation:** The width of the paperback book is  $4\frac{1}{4}$  inches. Round this measurement to the nearest inch.
- **b. Geometry:** An octagon has how many more sides than a pentagon?
- c. Money: What coin is 10% of a dollar?
- d. Number Sense: 100 ÷ 4
- e. Number Sense: 100 ÷ 5
- **f. Estimation:** Fiona measured the width of the paper as  $21\frac{6}{10}$  cm. Round this measurement to the nearest centimeter.
- **g. Time:** Carmen's younger brother is 2 years 8 months old. How many months old is her brother?
- **h. Calculation:**  $10 \times 10$ ,  $\div 2$ , -1,  $\div 7$ , -1,  $\div 2$ , -1,  $\div 2$

# problem solving

Choose an appropriate problem-solving strategy to solve this problem. Freddy used a loop of string to form the rectangle shown at right. If Freddy uses the same loop to form a square, what will be the length of each side?

8 in.	
	4 in.

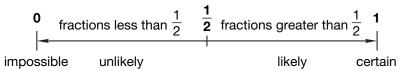
# **New Concept**

There are many situations whose future outcomes are uncertain. For example, the weather forecast might say that rain is likely tomorrow, but this would only be an educated guess. It might rain or it might not rain. If we take an airplane flight, we might arrive early, we might arrive late, or we might arrive on time. We cannot know for sure in advance.

#### **Math Language**

An event is an outcome (or group of outcomes) in a probability experiment.

**Probability** is a measure of *how likely* it is that an event (or combination of events) will occur. Probabilities are numbers between 0 and 1. An event that is **certain** to occur has a probability of 1. An event that is **impossible** has a probability of 0. If an event may or may not occur, then its probability is a fraction between 0 and 1. The more *likely* an event, the closer its probability is to 1. The more *unlikely* an event, the closer its probability is to 0. The diagram below uses words to describe the range of probabilities from 0 to 1.



Probabilities can be expressed as fractions, decimals, or percents.

The word **chance** is also used to describe the likelihood of an event. Chance is often expressed as a percent ranging from 0% (for events that are impossible) to 100% (for events that are certain to happen). If the chance of rain is forecast as 80%, then in the meteorologist's informed opinion, it is likely to rain.

The expression "50-50 chance" means an event is equally likely to happen (50%) as it is not to happen (50%). Added together, the chances (or probability) of an event happening or not happening total 100% (or 1). For example, if the chance of rain is 80%, then the chance that it will not rain is 20%. If the probability of winning a drawing is  $\frac{1}{1000}$ , then the probability of not winning the drawing is  $\frac{999}{1000}$ .

#### Example 1

A standard dot cube is rolled once. Which word best describes each event in parts a-d: certain, likely, unlikely, or impossible?



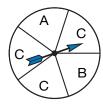
- a. The cube will stop with 3 dots on top.
- b. The cube will stop with more than 2 dots on top.
- c. The cube will stop with fewer than 7 dots on top.
- d. The cube will stop with more than 6 dots on top.
- **a. Unlikely.** There are six faces and only one has 3 dots. We would expect the cube to stop with 3 dots on top less than half the times the cube is rolled.

- **b.** Likely. Of the six faces on the dot cube, four have more than 2 dots. We would expect that a number greater than 2 would end up on top more than half the times the cube is rolled.
- c. Certain. All the faces have fewer than 7 dots, so every time the cube is rolled, the upturned face will have fewer than 7 dots.
- **d. Impossible.** None of the faces have more than 6 dots, so it is not possible for an upturned face to have more than 6 dots.

Many **experiments** involve probability. Some experiments that involve probability are tossing a coin, spinning a spinner, and selecting an object from a set of objects without looking. The possible results of such experiments are called **outcomes**. The probabilities of the outcomes of any experiment always add up to 1.

#### **Example 2**

The circle below is divided into 5 equal-sized sectors. Each sector is labeled by one of these letters: A, B, or C. Suppose the spinner is spun and stops in one of the sectors.



Find the probability of each of the possible outcomes A, B, and C.

The probability that the spinner will stop in a given sector is equal to that sector's fraction of the circle. Since outcome A corresponds to  $\frac{1}{5}$  of the whole, the probability that the spinner will stop in sector A is  $\frac{1}{5}$ . Outcome B also has a probability of  $\frac{1}{5}$ . Since outcome C corresponds to  $\frac{3}{5}$  of the whole, it has a probability of  $\frac{3}{5}$ . Notice that  $\frac{1}{5} + \frac{1}{5} + \frac{3}{5} = \frac{5}{5} = 1$ . (The probabilities of the outcomes of an experiment always total 1.)

### Example 3

A bag contains 5 red marbles, 3 blue marbles, and 2 yellow marbles. Suppose we pick one marble from the bag without looking.

- a. Find the probability that the marble is blue.
- b. Find the probability that the marble is not blue.

- **a.** The probability that we picked a blue marble is a fraction between 0 and 1. This fraction describes the number of blue marbles as a part of the overall group of marbles. There are 10 marbles, so there are 10 possible outcomes. Since 3 out of 10 marbles are blue, the probability that we picked a blue marble is  $\frac{3}{10}$ .
- **b.** The remaining 7 marbles are not blue, so the probability that the marble is not blue is  $\frac{7}{10}$ .

**Verify** What is the sum of the probabilities of drawing a blue marble and drawing a marble that is not blue?

#### Example 4

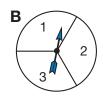
Ben spun a spinner 60 times and recorded the outcome shown in the table below. Refer to the table to answer the questions that follow.

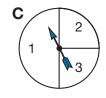
**Spinner Results** 

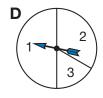
Sector Numbers	Number of Times
1	29
2	20
3	11

a. Which of these spinners most likely represents the spinner Ben used?







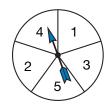


- b. If Ben spins the spinner 10 more times, about how many times is the spinner likely to stop in sector 1?
- a. We see that about half of the spins stopped in sector 1, so sector 1 is probably half of the face of the spinner. Therefore, we are choosing between spinners C and D. The spinner stopped in sector 2 about twice as many times as it stopped in sector 3. The best choice is spinner D.
- **b.** Since about half the spins have stopped in sector 1, we would expect the pattern to continue. The spinner is likely to stop **5 times** in sector 1.

## Lesson Practice

Use the spinner at right to answer problems a-d.

- **a.** What are all the possible outcomes?
- **b.** What is the probability that the spinner will stop on 3?



- **c.** What is the probability of spinning a number greater than three?
- d. What is the probability of spinning an even number?
- **e.** If the weather forecast states that the chance of rain is 40%, is it more likely to rain or not to rain?
- **f.** If today's chance of rain is 20%, then what is the chance that it will not rain today?
- **g.** Evaluate For the experiment described in Example 3, Seth said that the probability of picking a red marble was  $\frac{1}{2}$ . Do you agree or disagree with Seth? Why?
- **h. Multiple Choice** Refer to the table in Example 4. Which fraction best names the probability that the spinner will stop in sector 3?
  - **A**  $\frac{1}{2}$
- **B**  $\frac{1}{3}$
- **c**  $\frac{1}{4}$
- **D**  $\frac{1}{6}$

## **Written Practice**

Distributed and Integrated

- **1.** A foot equals 12 inches. A person who is 5 feet 4 inches tall is how many inches tall?
- 2. How many years is 10 centuries?
  - $\mathbf{3.}$  What word is used to name the perimeter of a circle?
  - \*4. Represent Use words to name the mixed number  $10\frac{7}{10}$ .
- \*5. Represent How many minutes is two thirds of an hour? Draw a diagram to illustrate and solve the problem.
  - **6.** Mr. Rohas heard the alarm go off at 6 a.m. and got up quickly. If he had fallen asleep at 11 p.m. the previous evening, how many hours of sleep did he get?

- 7. If 4 is the divisor and 12 is the quotient, then what is the dividend?
- \*8. What is the value of the place held by the zero in 321,098,765?
- **9.** List Which factors of 15 are also factors of 20?
- **10.** Assume that the sides of this regular hexagon are 3 cm long. Use a formula to find the perimeter of the hexagon.



11. 
$$3\frac{2}{3} - \left(2\frac{1}{3} + 1\frac{1}{3}\right)$$

**12.** 
$$3\frac{1}{3} + \left(2\frac{2}{3} - 1\frac{1}{3}\right)$$

**15.** Which number in this problem is the divisor?

$$6 \div 3 = 2$$

**21.** 
$$\frac{10}{10} - \frac{9}{10}$$

**22.** 
$$4\frac{2}{3} - \frac{1}{3}$$

**23.** 
$$5\frac{2}{2} - 1\frac{1}{2}$$

- **24.** From May 1 of one year to August 1 of the next year is how many months?
- \*25. Juan's last class of the afternoon begins 2 hours 20 minutes after the time shown on the clock. At what time does Juan's last class begin?



- **26. a.** How many years is a millennium?
  - **b.** How many years is half of a millennium?
  - **c.** Write a fraction equal to  $\frac{1}{2}$  using the numbers in the answers to parts **a** and **b**.
- \*27. If a standard dot cube is rolled once, what is the probability that it will land with more than one dot on top?
- **28.** Nimeesha's first three test scores were 80, 80, and 95. What was the average of Nimeesha's first three test scores?
- **29.** The multiple-choice question listed four choices for the answer. Kyla figured she had a 25% chance of guessing the correct answer. What was her chance of not correctly guessing the answer?
- **30.** When Leif turned 10, his mom was four times his age. How old will she be when Leif turns 15?



Real-World Connection Eli spun a spinner 40 times and recorded the outcomes in the table below:

#### **Spinner Results**

Sector Number	Number of Times
1	5
2	20
3	10
4	5

- **a.** Use the data in the table to draw a spinner that represents the spinner used.
- **b.** If Eli spins the spinner 10 more times, then how many times is the spinner likely to stop in sector 2?



## Writing Quotients with Mixed Numbers



#### facts

### mental math

### Power Up G

- **a. Estimation:** Round  $18\frac{5}{8}$  in. to the nearest inch.
- **b. Estimation:** Round  $12\frac{3}{8}$  in. to the nearest inch.
- **c. Estimation:** Round  $4\frac{1}{16}$  in. to the nearest inch.
- **d. Number Sense:** How much is  $800 \div 10? \dots 800 \div 20? \dots 800 \div 40?$
- e. Percent: 50% of 800
- **f. Number Sense:**  $3\frac{1}{2} + 3\frac{1}{2}$
- **g. Measurement:** One pound equals 16 ounces. Vanessa bought  $1\frac{1}{2}$  pounds of bananas. How many ounces did the bananas weigh?
- **h. Fractional Parts:** Myra spends  $\frac{1}{3}$  of each 24-hour day sleeping. How many hours does Myra sleep each day?

# problem solving

Choose an appropriate problem-solving strategy to solve this problem. Audra purchased two books at the school book fair: a mystery novel and a science fiction novel. The length of the mystery novel was 192 pages, and the length of the science fiction novel was 128 pages. Audra read 32 pages each day. Assuming she finished one book before starting the other, how much longer did it take Audra to read the mystery novel than the science fiction novel?

## New Concept

As we saw in Lessons 40 and 43, we sometimes need to write a division answer as a mixed number. In the problem on the next page, we do this by writing the remainder as a fraction.

If two children share 5 dumplings equally, how many dumplings will each receive?

We divide 5 into 2 equal parts. We find that the quotient is 2 and the remainder is 1; each child 2)5 will receive 2 dumplings, and there will be 1 extra dumpling. We can take the extra dumpling and divide it in half. Then each child will receive  $2\frac{1}{2}$  dumplings.

To write a remainder as a fraction, we simply make the remainder the numerator of the fraction and make the divisor the denominator of the fraction.

**Connect** If two people share \$5.00 equally, what amount of money will each person receive?

### Example 1

Divide: 3)50. Write the quotient as a mixed number.

We divide and find that the remainder is 2. We make the remainder the numerator of the fraction, and we make the divisor the denominator of the fraction. The quotient is  $16\frac{2}{3}$ .

$$\begin{array}{c|c}
16\frac{2}{3} \\
3)50 \\
\frac{3}{20} \\
\frac{18}{2}
\end{array}$$

### **Example 2**

A 15-foot-long board is cut into 4 equal lengths. How long is each length?

We divide 15 feet by 4 and find that the quotient is not a whole number of feet. The quotient is more than 3 feet but less than 4 feet; it is 3 feet plus a fraction. To find the fraction, we write the remainder as the numerator of the fraction and write the divisor as the denominator of the fraction. We find that the length of each piece of wood is  $3\frac{3}{4}$  feet.

Analyze How many inches long is each of the four boards? Explain your thinking.

### Example 3

A group of four friends collected aluminum cans and received \$21 from a recycling center for the cans. Each friend received an equal share of the money. Which quotient represents the number of dollars each friend received?

Since we can divide \$21 into four equal parts, each friend received  $5\frac{1}{4}$  dollars.

**Connect** How do we express  $5\frac{1}{4}$  dollars as dollars and cents?

In the lesson practice that follows, we will continue to write quotients with remainders, unless a problem asks that the answer be written with a fraction.

### Lesson Practice

Divide. Write each quotient as a mixed number.

**c.** 
$$\frac{16}{5}$$

**f.** 
$$\frac{49}{10}$$

i. 
$$\frac{31}{8}$$

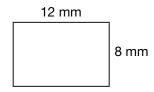
## Written Practice

Distributed and Integrated

- 1. Cesar bought 8 baseball cards for 35 cents each. If he paid with a \$5 bill, how much should he have received in change?
- 2. Davu bought a 21-inch ribbon. She cut it into 4 equal lengths. How long was each ribbon? Write the answer as a mixed number.
- **Represent** Draw a diagram to illustrate and solve this problem: T'Leesha used  $\frac{3}{5}$  of a sheet of stamps to mail cards. If there are 100 stamps in a whole sheet, then how many stamps did T'Leesha use? What percent of the stamps did T'Leesha use?
  - **4.** Round 1776 to the nearest hundred.

- **\*5. Multiple Choice** In which of these numbers does the 5 have a value of 500,000?
  - **A** 186,542,039
- **B** 347,820,516
- **C** 584,371,269
  - **D** 231,465,987

**6.** What is the perimeter of this rectangle?



9. 
$$50w = 800$$

**10.** 
$$1400 + m = 7200$$

11. 
$$$1.25 \times 80$$

**16.** 
$$\frac{4228}{7}$$

**18.** 
$$\frac{5}{5} - \frac{1}{5}$$

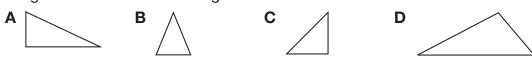
**19.** 
$$3\frac{1}{3} - \frac{1}{3}$$

**20.** 
$$4\frac{6}{6} - 2\frac{5}{6}$$

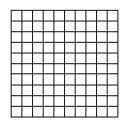
- **21.** Divide: 3)62. Write the quotient as a mixed number.
- **22.** What is the denominator of the fraction in  $6\frac{3}{4}$ ?
- **23.** In a division problem, if the divisor is 3 and the quotient is 9, then what is the dividend?
- **24.** What year was five centuries before 1500?
- **25.** If the radius of this circle is 12 millimeters, then what is the diameter of the circle?



- \*26. Predict There are 2 red marbles, 3 blue marbles, and 6 yellow marbles in a bag. If Maureen takes one marble from the bag without looking, what is the probability that the marble will be red?
  - **27. Multiple Choice** Which of these triangles appears to be both a right triangle and an isosceles triangle?



**28.** Analyze The large square has been divided into 100 smaller squares. How many small squares equal  $\frac{3}{4}$  of the large square?



- **29.** Represent China has the largest population of all the countries in the world. In the year 2002, there were approximately one billion, two hundred eighty-four million, two hundred four thousand people living in China. Use digits to write the approximate number of people living in China.
- 30. Sharell bought 2 gallons of milk. She also bought a box of cereal that cost \$3.48. If she paid for the 3 items with a \$10 bill and received \$0.32 in change, then what was the price of each gallon of milk? Explain how you found your answer.



The school choir is having a car wash to raise \$750 to buy new songbooks. Each car wash will cost \$3.50.

- a. If they wash 228 cars, how much money will they raise?
- **b.** Is this more than or less than their goal?
- c. Explain how you found the answer to part a.



## Subtracting a Fraction from 1



#### facts

### mental math

Power Up F

- **a. Estimation:** Round  $5\frac{3}{16}$  in. to the nearest inch.
- **b. Percent:** In bowling, the highest possible score for one game is 300. D'Shaun scored 50% of 300. What was his score?
- **c. Geometry:** Altogether, how many sides are on 4 dozen squares?
- d. Time: How many days are in 3 weeks 3 days?
- e. Fractional Parts: Half of 101
- f. Percent: 10% of 50
- **g. Money:** Shaquana saved \$35 each month for six months. How much money did she save?
- **h. Calculation:**  $6 \times 6$ , -1,  $\div 7$ ,  $\times 4$ , +1,  $\div 7$

# problem solving

Choose an appropriate problem-solving strategy to solve this problem. Two rolls of pennies have a total value of \$1.00. Paul opened up two rolls of pennies and formed a square array with the pennies. How many pennies were in each column of the array?

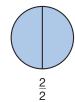
## New Concept

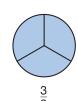
#### **Thinking Skill**

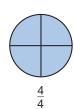
Discuss

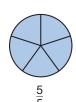
Why does  $\frac{4}{4} = 1$ ?

We know that two halves make a whole. Similarly, it takes **three** thirds or **four** fourths or **five** fifths to make one whole.









We see that each of these is a "whole pie," yet we can use different fractions to name each one. Notice that the numerator and the denominator are the same when we name a "whole pie." This is a very important idea in mathematics. Whenever the numerator and denominator of a fraction are equal (but not zero), the fraction is equal to 1.

### **Example 1**

Write a fraction equal to 1 that has a denominator of 4.

A fraction equal to 1 that has a denominator of 4 would also have a numerator of 4, so we write  $\frac{4}{4}$ .

Analyze Since  $\frac{4}{4}$  equals 1, what does  $\frac{5}{4}$  equal? Explain how you know.

### **Example 2**

For a Saturday night snack, Bailey and her friends cooked two identical pizzas. Bailey ate  $\frac{1}{4}$  of one pizza and her friends ate  $\frac{3}{4}$  of the other pizza. What amount of pizza was eaten?

We can show this problem with fraction manipulatives or by drawing a picture that represents each part.



We add and find that the sum is  $\frac{4}{4}$ . We should always write our answers in simplest form. The simplest name for  $\frac{4}{4}$  is **1.** We found that one fourth plus three fourths is equal to one.

$$\frac{1}{4} + \frac{3}{4} = \frac{4}{4} = 1$$

## **Example 3**

Compare:  $4\frac{3}{3} \bigcirc 5$ 

The mixed number  $4\frac{3}{3}$  means  $4+\frac{3}{3}$ . Since  $\frac{3}{3}$  equals 1, the addition  $4+\frac{3}{3}$  is the same as 4+1, which is 5. We find that  $4\frac{3}{3}$  and 5 are equal.

$$4\frac{3}{3} = 5$$

**Justify** Explain why  $4\frac{4}{3}$  is greater than 5.

#### **Example 4**

One morning during summer vacation, Mason spent  $1\frac{1}{2}$  hours cleaning out the garage and his sister Alyssa spent  $1\frac{1}{2}$  hours doing yard work. How many hours did Mason and Alyssa work that morning altogether?

We add and find that the sum is  $2\frac{2}{2}$ . The mixed number  $2\frac{2}{2}$  means  $2+\frac{2}{2}$ . Since  $\frac{2}{2}$  equals 1, the addition  $2+\frac{2}{2}$  is the same as 2+1, which is **3 hours.** 

$$1\frac{1}{2} + 1\frac{1}{2} = 2\frac{2}{2} = 3$$

**Discuss** Explain how to find the sum of  $\frac{3}{2} + \frac{3}{2}$ .

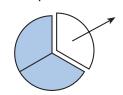
To subtract a fraction from 1, we rewrite 1 as a fraction. There are many fractions equal to 1, such as  $\frac{2}{2}$ ,  $\frac{3}{3}$ ,  $\frac{6}{6}$ , and  $\frac{10}{10}$ . We look at the fraction that is subtracted to decide which name for 1 we should use.

#### **Example 5**

Ja'Von baked a blueberry pie. After dinner, he and his family ate  $\frac{1}{3}$  of the pie for dessert. What fraction of the pie was not eaten?

We can show this problem with fraction manipulatives or by drawing a picture that represents a whole pie. If we remove one third of the pie, how much of the pie is still in the pan?

Before we can remove a third, we first slice the pie into three thirds. Then we can subtract one third. We see that two thirds of the pie is still in the pan. Using pencil and paper, we rewrite 1 as  $\frac{3}{3}$ . Then we subtract.



$$1 - \frac{1}{3}$$

$$\downarrow \qquad \qquad \downarrow$$

$$\frac{3}{3} - \frac{1}{3} = \frac{2}{3}$$

We could have chosen any name for 1, such as  $\frac{2}{2}$  or  $\frac{4}{4}$  or  $\frac{3682}{3682}$ , but we chose  $\frac{3}{3}$  because it has the same denominator as the fraction that was subtracted. Remember, we can only add and subtract fractions when their denominators are the same.

373

## Lesson Practice

**a.** Write a fraction equal to 1 that has a denominator of 3.

Compare:

**b.** 
$$\frac{4}{4}$$
 1

**c.** 
$$5\frac{4}{4}$$
  $\bigcirc$  6

Add:

**d.** 
$$\frac{3}{10} + \frac{7}{10}$$

**e.** Use fraction manipulatives to add  $3\frac{3}{5}+2\frac{2}{5}$ . Explain your solution using words.

Subtract:

**f.** 
$$1 - \frac{1}{4}$$

g. 
$$1 - \frac{2}{3}$$

h. How many fraction names for 1 are there?

## Written Practice

Distributed and Integrated

- **1.** Cynna jumped rope for 3 minutes 24 seconds without stopping. How many seconds are in 3 minutes 24 seconds?
- **2.** Brady's mom baked 5 dozen breadsticks, and Brady ate one tenth of them. How many breadsticks did he eat?
- \*3. Represent Draw a quadrilateral that has a pair of horizontal, parallel line segments of different lengths.
  - 4. List Which factors of 8 are also factors of 20?
- **5.** How many seconds is two fifths of a minute? Two fifths of a minute is what percent of a minute?
  - **6.** Seventeen sketches are to be displayed on three bulletin boards. Is it possible for each bulletin board to display the same number of sketches? Explain why or why not.

7. 
$$\frac{1}{4} + \frac{3}{4}$$

8. 
$$1\frac{1}{3} + 2\frac{2}{3}$$

9. 
$$2\frac{5}{8} + \frac{3}{8}$$

**10.** 
$$1-\frac{1}{4}$$

11. 
$$1-\frac{3}{8}$$

**12.** 
$$2\frac{8}{8} - \frac{3}{8}$$

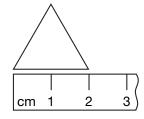
- **16.** Represent Use words to name the mixed number  $8\frac{9}{10}$ .
- 17. Divide:  $\frac{15}{4}$ . Write the quotient as a mixed number.

For problems 18 and 19, write the answer with a remainder.

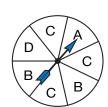
**19.** 60) 850

**21.** 507 × \$3.60

- \*23. Multiple Choice Which of these mixed numbers is *not* equal to 3?
  - **A**  $2\frac{3}{3}$
- **B**  $3\frac{2}{2}$  **C**  $2\frac{4}{4}$
- **D**  $2\frac{8}{9}$
- **24.** Write a fraction equal to 1 that has a denominator of 5.
- **25.** What is the perimeter of this equilateral triangle?



- \*26. Analyze To multiply 35 by 21, Germaine thought of 21 as 20 + 1 and performed the multiplication mentally. Show two ways Germaine could multiply the numbers. Which way do you think is easier to perform mentally? Why?
  - 27. The face of this spinner is divided into equal sectors. Refer to the spinner to answer parts a and b.

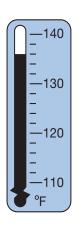


- a. Which two outcomes are equally likely?
- **b.** What is the probability that the spinner will stop on C?

375

\*28. A teacher asked 19 fifth grade students to state the number of children in their family. Their responses made up this data set:

- a. Make a line plot to display this data.
- **b.** What is the median?
- c. What is the mode?
- 29. During the 2005 baseball season, the Colorado Rockies team had 1477 hits. The Chicago Cubs team had 1506 hits. What is a reasonable estimate of the number of hits those teams had that year altogether? Explain why your estimate is reasonable.
- **30.** The highest temperature ever recorded on the continent of Africa is shown on this thermometer. What was that temperature?





Mrs. Hernandez's fifth grade class studied the first settlers to their state and decided to make a quilt as an art project. When finished, the quilt will be nine small squares (all the same size) joined to make one large square. The class has completed 4 squares.

- **a.** Draw a square made of 9 smaller squares to represent the quilt and shade the number of parts that have been completed.
- **b.** Write a fraction naming the part of the quilt that is finished.
- **c.** Write a fraction naming the part of the quilt that is not finished.
- **d.** Write an equation to show that the sum of the completed parts and the parts that are not completed equals the whole quilt.



## Finding a Fraction to Complete a Whole



#### facts

### mental math

### Power Up G

**a. Number Sense:**  $\frac{1}{2}$  plus what fraction equals 1?

**b. Number Sense:**  $\frac{1}{3}$  plus what fraction equals 1?

**c.** Number Sense:  $\frac{1}{4}$  plus what fraction equals 1?

**d. Number Sense:**  $\frac{1}{8}$  plus what fraction equals 1?

**e. Number Sense:** How much is 900 ÷ 10? ... 900 ÷ 30? ... 900 ÷ 90?

**f. Money:** 9 × 25¢

g. Percent: 25% of a dozen

**h. Calculation:**  $9 \times 9, -1, \div 2, -1, \div 3$ 

# problem solving

Choose an appropriate problem-solving strategy to solve this problem. Deneka's lacrosse team plays 15 games every season. Last season, the team won only half as many games as they won this season. This season, the team won twice as many games as they lost. What was the team's win-loss record this season? What was the team's win-loss record last season?

## **New Concept**

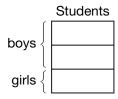
Sometimes we are given one part of a whole and need to know the other part of the whole. Consider this word problem:

One third of the students are girls. What fraction of the students are boys?

We answer problems like this by thinking of the entire group as a whole. We can draw a rectangle to represent the whole group of students. The problem states that  $\frac{1}{3}$  of the students are girls, so we divide the rectangle into three parts and label one of the parts "girls."



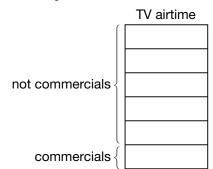
The fraction of the students that is not girls must be boys. Since the girls make up 1 of the 3 parts, the boys must make up 2 of the 3 parts. Thus, two thirds of the students are boys.



#### **Example 1**

Taye found that commercials make up one sixth of TV airtime. What fraction of TV airtime is not commercials?

We begin by thinking of TV airtime as a whole. We draw a rectangle to show this. The problem states that  $\frac{1}{6}$  of the airtime is made up of commercials. So we divide the rectangle into six equal parts. We label one part "commercials." We see that the fraction of TV airtime that is not commercials is  $\frac{5}{6}$ .



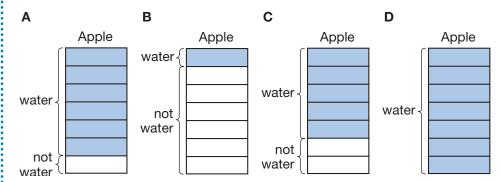
#### **Thinking Skill**

#### Justify

Explain how we can check the answer.

### Example 2

In his science fair project, Nelson found that  $\frac{6}{7}$  of the mass of an apple is water. Which diagram below matches this information?



The correct diagram is divided into seven equal parts with six parts identified as water, so the correct diagram is **A**.

## Lesson Practice

- **a.** Laxmi has read one fourth of her book. What fraction of her book is left to read?
- **b.** Five eighths of the gymnasts were able to do a back handspring. What fraction of the gymnasts were unable to do a back handspring?
- **c.** If three fifths of the spectators were rooting for the home team, then what fraction of the spectators were not rooting for the home team?

## Written Practice

Distributed and Integrated

- **1.** In one class there are three more girls than boys. There are 14 boys. How many students are in the class?
- **2.** Calvin bought two bicycle tubes for \$2.39 each and a tire for \$4.49. The tax was 56¢. If he paid \$10, how much money should he get back in change?
- 3. How many decades passed between the years 1800 and 1900?
  - **4.** The diameter of Stella's bicycle wheel is 24 inches. What is the radius of the wheel?

- \*5. Round 487 and 326 to the nearest hundred. Then add the rounded numbers. What is the sum?
- **6.** Find each missing numerator:

**a.** 
$$\frac{1}{7} = 1$$

**b.** 
$$4 = 3\frac{\Box}{4}$$

- \*7. When Mya was born, she weighed 7 pounds 12 ounces. How many ounces did Mya weigh at birth? (One pound equals 16 ounces.)
- **8.** What is the perimeter of this square?



9. 
$$\frac{1}{6} + \frac{2}{6} + \frac{3}{6}$$

**10.** 
$$3\frac{3}{5} + 1\frac{2}{5}$$

11. 
$$1-\frac{1}{8}$$

**12.** 
$$4\frac{5}{5} - 1\frac{2}{5}$$

14. 
$$\frac{$36.72}{9}$$

- **15.** Divide:  $\frac{23}{10}$ . Write the quotient as a mixed number.
- **16.** Analyze Adrian found that commercials made up one eighth of radio airtime. What fraction of radio airtime was not commercials? What percent of radio airtime was commercials?

**17.** 
$$374 \times 360$$

**19.** 
$$60 \times (800 \div 40)$$

- **\*21.** Compare:  $\frac{4}{4} \bigcirc \frac{5}{5}$ 
  - **22.** Write a fraction equal to 1 that has a denominator of 8.
  - **23.** Connect To what fraction is the arrow pointing?



- 24. Anahi begins work each morning at 7:30 a.m. She takes a lunch break at 11:25 a.m. and returns to work at noon. How long is Anahi's lunch break?
- **25.** One marble is selected from a bag containing 2 red marbles, 5 green marbles, and 6 white marbles.
  - a. What fraction describes the probability that the marble is green?
  - **b.** What fraction describes the probability that the marble is not green?
- \*26. Multiple Choice Which of these division problems will not result in a remainder?
  - **A**  $321 \div 2$
- **B**  $421 \div 3$  **C**  $521 \div 6$  **D**  $621 \div 9$
- **27.** Refer to the information below to answer parts **a** and **b**.

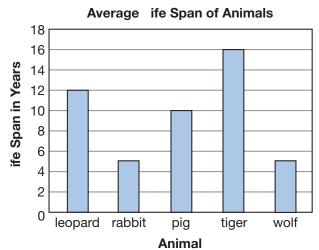
Kabira has 30 CDs. Not every CD has the same number of songs on it. This table shows how many of Kabira's CDs have 9, 10, 11, 12, 13, or 14 songs.

**Frequency Table** 

Songs per CD	Frequency
9	1
10	4
11	7
12	13
13	3
14	2

- a. Among Kabira's CDs, what number of songs does a CD most frequently have?
- **b.** How many of Kabira's CDs have more than 10 songs?

**28.** Use the bar graph to answer parts **a** and **b**.



- **a.** How does the average life span of a tiger compare to the average life span of a leopard?
- **b.** The average life span of a white rhinoceros is four times longer than the life span of a rabbit. What is the average life span of a white rhinoceros?
- 29. Multiple Choice Roberto is 10 years old. Fabian is 3 years younger than Yasmin. Yasmin is half as old as Roberto. Which number sentence could be used to find e, Fabian's age?

**A** 
$$(10 \times 2) - 3 = e$$

**B** 
$$(10 \div 2) + 3 = e$$

**C** 
$$(10 \times 2) + 3 = e$$

**D** 
$$(10 \div 2) - 3 = e$$

30. A wheel on Grady's bicycle travels about 78 inches each time it goes around once. About how many inches does the wheel travel if it goes around 50 times? Explain why your estimate is reasonable.



Dion puts seven tenths of his allowance into his savings account each month. What fraction of his allowance does *not* go into his account? Draw a diagram to solve the problem.

## Focus on

## Line Graphs

Often we are interested in seeing the changes in data that occur over a period of time. Below we show the average temperature in the city of Boston for each month of the year.

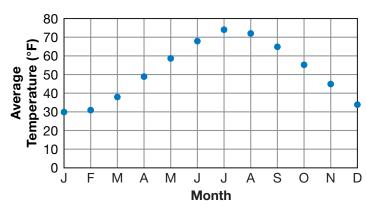
**Average Boston Temperature** 

Month	Temp.
January	30°F
February	31°F
March	38°F
April	49°F
May	59°F
June	68°F

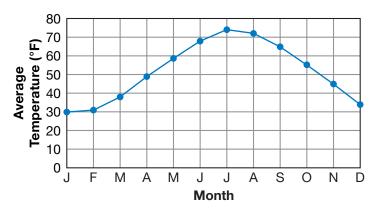
Month	Temp.
July	74°F
August	72°F
September	65°F
October	55°F
November	45°F
December	34°F

The temperature is lowest in January and February. Then the weather warms up steadily until summer arrives. It stays warm through August and then cools steadily after that. In December the temperature is almost as low as it is at the beginning of the year.

To show the change of temperature over time, we can use a line graph. We will draw the **line graph** on a grid. First we label each of the 12 months along the grid's **horizontal axis**. Then we label temperatures from 0°F through 80°F along the grid's **vertical axis**. We label up to 80° on the grid because we need to graph temperatures as high as 74°. We choose our interval to be 10°F on the vertical axis. We could use a smaller interval instead (such as 5°), but then our grid would be bigger. Above each month, we place a dot at a height equal to the normal temperature for that month.



Finally, we connect the dots with line segments to produce our line graph. The rising line from January until July shows that the temperature is increasing. The falling line from July to December shows that the temperature is decreasing. The line is steepest in spring and in fall. During these times, the average temperature is changing the most quickly.



- **1. Interpret** In what month was the highest average temperature, and what was that temperature?
- **2.** What is the range of the average temperatures shown in the line graph?
- **3.** From March through June, the average temperature increases about how many degrees per month?

Every two months Liz weighed Jake, her Labrador retriever, and recorded the weight in a table.

Age	Birth	2 mo	4 mo	6 mo	8 mo	10 mo	12 mo
Weight (in kg)	0.5	6	12	17	21	24	27

- **4. Represent** Make a line graph showing Jake's weight over this 12-month period.
- **5. Multiple Choice** In which time period did Jake's weight double?

**A** 2 mo-4 mo

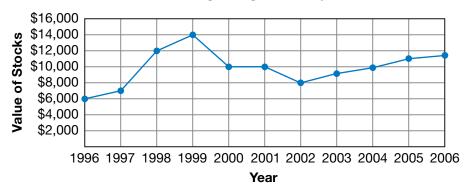
**B** 4 mo-6 mo

**C** 6 mo-8 mo

**D** 4 mo-8 mo

**6.** Predict Jake's weight at 14 months.

Mr. Escobar invests in stocks. He has constructed a line graph to show the value of his stocks at the beginning of each year.



- **7.** How much were Mr. Escobar's stocks worth at the beginning of 2001? About how much were the stocks worth at the beginning of 2003?
- **8.** At the beginning of which year were his stocks worth the most? About how much were they worth then?
- **9.** During which year did his stocks increase in value the most? About how much was the increase?
- **10.** During which year did the value of his stocks decrease the most? About how much was the decrease?
- **11.** Estimate the overall change in the value of his stocks from the beginning of 1996 to the beginning of 2006.

# Activity

## Making a Line Graph

Material needed:

Lesson Activity 31

Kelp has some of the most remarkable growth rates in the plant kingdom. Off the coast of southern California, one variety of kelp can grow 30 centimeters per day. Complete the table below, and then make a line graph showing the possible growth of the kelp over the period of a week.

Days	1	2	3	4	5	6	7
Growth (in cm)	30	60	90				



- **a.** Using the Internet or an almanac, find the population of your state in the years 1950, 1960, 1970, 1980, 1990, and 2000. Make a line graph showing the population change by decades.
- b. Explain The table shows the average life span of a Federal Reserve Note. Choose the best type of graph to display the data (pictograph, bar graph or line graph). Explain your choice. Then make the graph you chose.

Average Life Span of a Federal Reserve Note (in months)

Federal Reserve Note	Average Life Span (in months)
\$1 bill	22
\$5 bill	24
\$10 bill	18
\$20 bill	25
\$50 bill	55
\$100 bill	60

**c.** The chart shows the number of endangered birds in the United States between 1980 and 2005. Display the data in a line graph. Write a question that can be answered by using the data in your graph.

Number of Endangered Birds in United States Between 1980 and 2005

Year	Number of Endangered Birds in United States
1980	58
1985	68
1990	72
1995	75
2000	78
2005	77