

• Reducing Fractions, Part 1

Power Up

facts

Power Up H

estimation

Hold your hands about one foot apart. Hold your hands about one yard apart.

mental math

- a. **Measurement:** One mile is how many feet?
- b. **Fractional Parts:** $\frac{1}{4}$ of 30
- c. **Fractional Parts:** $\frac{1}{4}$ of 300
- d. **Powers/Roots:** 5^2
- e. **Time:** After school J'Vonte walks his dog for 30 minutes and then starts his homework. J'Vonte is halfway through his daily walk. How long before J'Vonte starts his homework?
- f. **Percent:** 10% of \$300
- g. **Estimation:** Choose the more reasonable estimate for the diameter of a CD: 12 centimeters or 12 millimeters.
- h. **Calculation:** $30 \times 30, + 100, \div 2, - 100, \div 4$

problem solving

Choose an appropriate problem-solving strategy to solve this problem. List the possible arrangements of the letters A, E, and R. What percent of the possible arrangements spell words?

New Concept

In Lesson 79, we practiced making equivalent fractions by multiplying by a fraction name for 1. We changed the fraction $\frac{1}{2}$ to the *equivalent fraction* $\frac{3}{6}$ by multiplying by $\frac{3}{3}$.

$$\frac{1}{2} \times \frac{3}{3} = \frac{3}{6}$$



Visit www.SaxonMath.com/Int5Activities for a calculator activity.

Multiplying by $\frac{3}{3}$ made the **terms** of the fraction greater. The terms of a fraction are the numerator and the denominator. The terms of $\frac{1}{2}$ are 1 and 2. The terms of $\frac{3}{6}$ are 3 and 6.

Generalize State a rule for writing an equivalent fraction using multiplication.

Sometimes we can make the terms of a fraction smaller by dividing by a fraction name for 1. Here we change $\frac{3}{6}$ to $\frac{1}{2}$ by dividing both terms of $\frac{3}{6}$ by 3:

$$\frac{3}{6} \div \frac{3}{3} = \frac{1}{2} \quad \begin{array}{l} (3 \div 3 = 1) \\ (6 \div 3 = 2) \end{array}$$

Math Language

Reducing a fraction is also referred to as writing a fraction in **lowest terms** or writing a fraction in **simplest form**.

Generalize State a rule for writing an equivalent fraction using division.

Changing a fraction to an equivalent fraction with smaller terms is called **reducing**. We reduce a fraction by dividing both terms of the fraction by the same number.

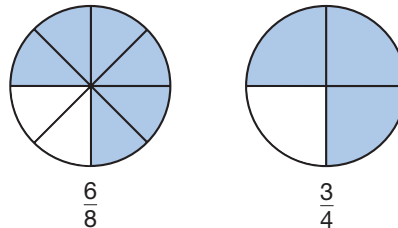
Example 1

Reduce the fraction $\frac{6}{8}$ by dividing both the numerator and the denominator by 2.

We show the reducing process below:

$$\frac{6 \div 2}{8 \div 2} = \frac{3}{4}$$

Model We can use fraction manipulatives to show equivalent fractions. The reduced fraction $\frac{3}{4}$ has smaller terms than $\frac{6}{8}$. We can see from the picture below, however, that $\frac{3}{4}$ and $\frac{6}{8}$ are equivalent fractions.



Not all fractions can be reduced. Only fractions whose terms can be divided by the same number can be reduced.

Example 2

Which of these fractions cannot be reduced?

A $\frac{2}{6}$

B $\frac{3}{6}$

C $\frac{4}{6}$

D $\frac{5}{6}$

We will consider each fraction:

A The terms of $\frac{2}{6}$ are 2 and 6. Both 2 and 6 are even numbers, so they can be divided by 2. The fraction $\frac{2}{6}$ can be reduced to $\frac{1}{3}$.

B The terms of $\frac{3}{6}$ are 3 and 6. Both 3 and 6 can be divided by 3, so $\frac{3}{6}$ can be reduced to $\frac{1}{2}$.

C The terms of $\frac{4}{6}$ are 4 and 6. Both 4 and 6 are even numbers, so they can be divided by 2. The fraction $\frac{4}{6}$ can be reduced to $\frac{2}{3}$.

D The terms of $\frac{5}{6}$ are 5 and 6. The only whole number that divides both 5 and 6 is 1. Since dividing by 1 does not make the terms smaller, the fraction $\frac{5}{6}$ cannot be reduced. The answer to the question is **D**.

Example 3

Add: $\frac{1}{8} + \frac{5}{8}$. Reduce the answer.

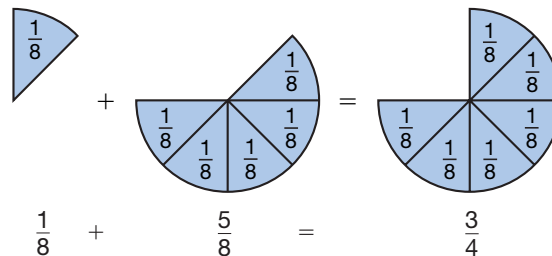
We add $\frac{1}{8}$ and $\frac{5}{8}$.

$$\frac{1}{8} + \frac{5}{8} = \frac{6}{8}$$

The terms of $\frac{6}{8}$ are 6 and 8. We can reduce $\frac{6}{8}$ by dividing each term by 2.

$$\frac{6 \div 2}{8 \div 2} = \frac{3}{4}$$

Model We can also use fraction manipulatives to show that the sum of $\frac{1}{8}$ and $\frac{5}{8}$ is $\frac{3}{4}$.



Example 4

Caroline has a box of beads that are all the same size and shape but are different colors. The box has 4 red beads, 6 yellow beads, and 20 blue beads. Without looking, Caroline chose one bead from the box.

- What are all the possible outcomes?
- What is the probability that the bead Caroline chose was blue?
 - There are three different colors of beads, so the possible outcomes are **red bead, yellow bead, and blue bead**.
 - Since 20 of the 30 beads are blue, the probability that Caroline chooses a blue bead is $\frac{20}{30}$. We reduce this ratio to $\frac{2}{3}$.

Example 5

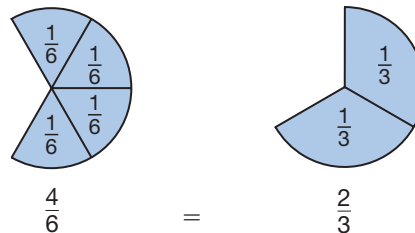
Subtract: $5\frac{5}{6} - 2\frac{1}{6}$. Reduce the answer.

First we subtract.

$$5\frac{5}{6} - 2\frac{1}{6} = 3\frac{4}{6}$$

Then we reduce $3\frac{4}{6}$. We reduce a mixed number by reducing its fraction.

Model We can use fraction manipulatives to reduce $3\frac{4}{6}$.



Since the fraction $\frac{4}{6}$ reduces to $\frac{2}{3}$, the mixed number $3\frac{4}{6}$ reduces to $3\frac{2}{3}$.

If an answer contains a fraction that can be reduced, we should reduce the fraction. Be aware of this as you work the problems in the problem sets.

Lesson Practice

- Reduce $\frac{8}{12}$ by dividing both 8 and 12 by 4.
- Multiple Choice** Which of these fractions *cannot* be reduced?

A $\frac{2}{8}$

B $\frac{3}{8}$

C $\frac{4}{8}$

D $\frac{6}{8}$

Add, subtract, or multiply as indicated. Remember to reduce your answers.

c. $\frac{3}{8} - \frac{1}{8}$

d. $\frac{3}{10} + \frac{3}{10}$

e. $\frac{2}{3} \times \frac{1}{2}$

f. In Example 4, what is the probability that Jenna chose a yellow bead?

Rewrite each mixed number with a reduced fraction:

g. $1\frac{3}{9}$

h. $2\frac{6}{9}$

i. $2\frac{5}{10}$

Find each sum or difference. Remember to reduce your answers.

j. $1\frac{1}{4} + 2\frac{1}{4}$

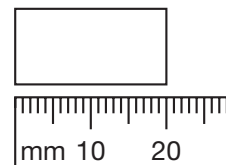
k. $1\frac{1}{8} + 5\frac{5}{8}$

l. $5\frac{5}{12} - 1\frac{1}{12}$

Written Practice

Distributed and Integrated

- (35) Evita's bowling scores for three games were 109, 98, and 135. Her highest score was how much more than her lowest score?
- (50) Find the average of the three bowling scores listed in problem 1.
- (74) Felix is 5 feet 4 inches tall. How many inches is 5 feet 4 inches?
- (68, 73) When twenty-six and five tenths is subtracted from thirty-two and six tenths, what is the difference?
- (79) **Analyze** Write a fraction equal to $\frac{2}{3}$ that has a denominator of 12. Then write a fraction equal to $\frac{1}{4}$ that has a denominator of 12. What is the sum of the two fractions you made?
- (80) **List** Write all the prime numbers between 20 and 30.
- (81) * Reduce the fraction $\frac{10}{12}$ by dividing both 10 and 12 by 2.
- (44, 53) 8. If the width of this rectangle is half its length, then what is the perimeter of the rectangle?



***9.** One fourth of the 24 members of an elementary school band can play more than one instrument. One half of the band members who can play more than one instrument also practice playing those instruments every day.

(46, 81)

- How many band members can play more than one instrument?
- How many band members who can play more than one instrument also practice every day?
- What fraction of the band members play more than one instrument and practice every day?

10. What is the area of the rectangle in problem 9?

(44, 72)

11. QS is 48 millimeters. Segment RS is half as long as QR . Find QR .

(61)



12. $3.4 + 6.25$

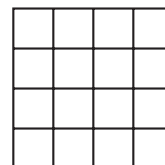
(73)

13. $6.25 - 3.4$

(73)

***14.** **Represent** The figure at right illustrates four squared (4^2). Using this model, draw a figure that illustrates three squared (3^2).

(78)



15. $6 \overline{) \$87.00}$

(34)

16. $40 \overline{) 2438}$

(54)

17. Divide 5280 by 9. Write the quotient as a mixed number with a reduced fraction.

(58)

18. $\$10 - (\$5.80 + 28\text{¢})$

(24, 70)

19. $5\frac{3}{5} + \left(4 - 1\frac{3}{5}\right)$

(59, 63)

***20.** Reduce: $\frac{3}{6}$

(81)

***21.** $\frac{4}{3} \times \frac{1}{2}$

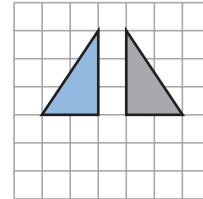
(76)

***22.** $\frac{10}{7} \times \frac{7}{10}$

(76)

- *23. Multiple Choice** Which transformation moves the blue triangle to the position of the gray triangle?
(Inv. 8)

A translation **B** rotation **C** reflection **D** slide



- *24.** Use this information to answer parts **a–b**:
(16, 21)

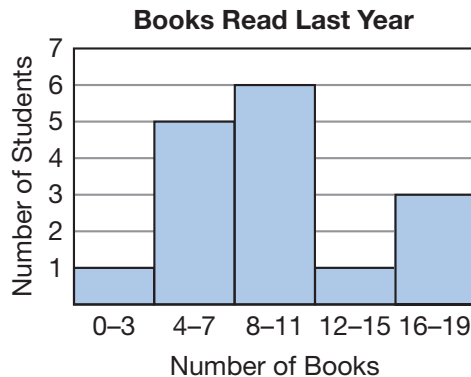
Rosa has a paper route. She delivers papers to 30 customers. At the end of the month, she gets \$6.50 from each customer. She pays the newspaper company \$135 each month for the newspapers.

- How much money does Rosa get each month from all her customers?
- How much profit does she make each month for her work?

- 25.** A standard number cube is rolled once.
(57)

- What is the probability that the upturned face is an even number?
- Describe a different event that has the same probability.

- *26.** The histogram below shows how many books some students read during the last year:
(Inv. 7)




- How many students read 12 books or more?
- How many students read 15 books or fewer?

- *27. Multiple Choice** Which of these Venn diagrams illustrates the relationship between rectangles (R) and squares (S)?
(45, Inv. 8)



28. Write 15% as a fraction. Then reduce the fraction by dividing both terms by 5.
(71, 81)

*29. Compare: $\frac{1}{2} \times \frac{1}{2} \bigcirc \frac{1}{2}$
(Inv. 2, 76)

30.  **Estimate** Parts of the shorelines of four Great Lakes form a national boundary between the United States and Canada.
(35, 49)

Shoreline	U.S. and Canada Length (miles)
Lake Superior	283
Lake Huron	261
Lake Erie	252
Lake Ontario	175

Shorelines Shared by U.S. and Canada	
Shoreline	Length (miles)
Lake Superior	283
Lake Huron	261
Lake Erie	252
Lake Ontario	175

Estimate the total length of the shorelines. Then explain why your estimate is reasonable.

• **Greatest Common Factor (GCF)**

Power Up

facts

Power Up H

mental math

In the expression $3(40 + 6)$, the sum of 40 and 6 is multiplied by 3. By using the *Distributive Property*, we can first multiply each addend and then add the partial products.

$$\begin{array}{c}
 \text{↖ ↗} \\
 3(40 + 6) \\
 120 + 18 = 138
 \end{array}$$

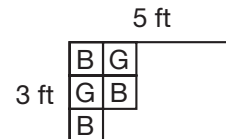
Use the Distributive Property to solve problems **a** and **b**.

- a. **Number Sense:** $3(20 + 7)$
- b. **Number Sense:** $4(30 + 6)$
- c. **Powers/Roots:** 6^2
- d. **Time:** What time is 30 minutes before 11:18 a.m.?
- e. **Number Sense:** Reduce the fractions $\frac{2}{4}$, $\frac{2}{6}$, $\frac{2}{8}$, and $\frac{2}{10}$.
- f. **Number Sense:** $\frac{1}{3}$ of 100
- g. **Measurement:** The classroom is 8 yards wide. How many feet is that?
- h. **Calculation:** $\sqrt{81}$, $+ 1$, $\times 5$, $- 2$, $\div 4$

problem solving

Choose an appropriate problem-solving strategy to solve this problem.

Marissa is covering a 5-by-3-foot bulletin board with blue and gold construction paper squares to make a checkerboard pattern. Each square is 1 foot by 1 foot. Copy this diagram on your paper, and complete the checkerboard pattern. What is the total area of the bulletin board? How many squares of each color does Marissa need?



New Concept

We have practiced finding the factors of whole numbers. In this lesson we will practice finding the **greatest common factor** of two numbers. The greatest common factor of two numbers is the largest whole number that is a factor of both numbers. The letters **GCF** are sometimes used to stand for the term *greatest common factor*.

To find the greatest common factor of 12 and 18, we first list the factors of each. We have circled the common factors; that is, the numbers that are factors of both 12 and 18.

Factors of 12: ①, ②, ③, 4, ⑥, 12

Factors of 18: ①, ②, ③, ⑥, 9, 18

The common factors are 1, 2, 3, and 6.

The greatest of these common factors is 6.

Example 1

Find the greatest common factor (GCF) of 8 and 20.

We will first find the factors and identify the common factors. The factors of 8 and 20 are listed below with the common factors circled.

Factors of 8: ①, ②, ④, 8

Factors of 20: ①, ②, ④, 5, 10, 20

We see that there are three common factors. The greatest of the three common factors is **4**.

We may use greatest common factors to help us reduce fractions.

Example 2

Use the GCF of 8 and 20 to reduce $\frac{8}{20}$.

In Example 1, we found that the GCF of 8 and 20 is 4. This means we can reduce $\frac{8}{20}$ by dividing both 8 and 20 by 4.

$$\frac{8 \div 4}{20 \div 4} = \frac{2}{5}$$

Lesson Practice

Find the greatest common factor (GCF) of each pair of numbers:

a. 6 and 9

b. 6 and 12

c. 15 and 100

d. 6 and 10

e. 12 and 15

f. 7 and 10

Reduce each fraction by dividing the terms of the fraction by their GCF:


g. $\frac{6}{9}$

h. $\frac{6}{12}$

i. $\frac{15}{100}$

Written Practice

Distributed and Integrated

1.  **Justify** ⁽⁴⁹⁾ Javier was paid \$34.50 for working on Saturday. He worked from 8 a.m. to 2 p.m. How much money did he earn per hour? Explain why your answer is reasonable.

2. ⁽⁶²⁾ Estimate the product of 396 and 507 by rounding to the nearest hundred before multiplying.

3. **Conclude** ⁽¹⁾ What is the next number in this counting sequence?

..., 3452, 3552, 3652, _____, ...

4. **Multiple Choice** ⁽⁷⁴⁾ Most adults are between 5 and 6 feet tall. The height of most cars is about

A 4 to 5 feet **B** 8 to 10 feet **C** 40 to 50 feet **D** 20 to 25 feet

5. ^(68, 73) When sixty-five and fourteen hundredths is subtracted from eighty and forty-eight hundredths, what is the difference?

6. ^(32, 53) If one side of a regular octagon is 12 inches long, then what is the perimeter of the octagon?

*7. **Multiple Choice** ⁽⁸⁰⁾ Which of these numbers is *not* a prime number?

A 11 **B** 21 **C** 31 **D** 41

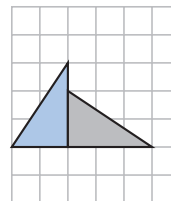
*8. ⁽⁸²⁾ a. Find the greatest common factor (GCF) of 20 and 30.

b. Use the GCF of 20 and 30 to reduce $\frac{20}{30}$.

9. ^(46, 74) How many inches is $\frac{3}{4}$ of a foot?

- *10. Multiple Choice** Which transformation moves the blue triangle to the position of the gray triangle?
(Inv. 8)

- A translation
B rotation
C reflection
D flip



- 11. a.** What number is $\frac{1}{3}$ of 12?
(Inv. 3)
b. What number is $\frac{2}{3}$ of 12?

***12.** Reduce: $\frac{6}{12}$
(81)

***13.** Compare: 2^3 \bigcirc 3^2
(78)

14. $\frac{5}{7} + \frac{3}{7}$
(75)

15. $\frac{4}{4} - \frac{2}{2}$
(59)

***16.** $\frac{2}{3} \times \square = \frac{6}{9}$
(79)

17. $\begin{array}{r} 976.5 \\ 470.4 \\ 436.7 \\ + 98.6 \\ \hline \end{array}$
(73)

18. $\begin{array}{r} \$40.00 \\ - \$32.85 \\ \hline \end{array}$
(13)

19. $\begin{array}{r} \$8.47 \\ \times 70 \\ \hline \end{array}$
(29)

20. $6 \overline{)43,715}$
(26)

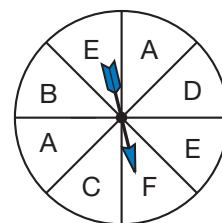
21. $\frac{2640}{30}$
(54)

22. $\begin{array}{r} 367 \\ \times 418 \\ \hline \end{array}$
(55)

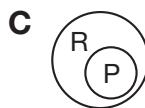
***23.** $3\frac{1}{4} + 3\frac{1}{4}$
(81)

24. $\$18.64 \div 4$
(26)

- *25. Analyze** Find the probability that with one spin, the spinner will *not* stop on A. Write the answer as a reduced fraction.
(57, 81)

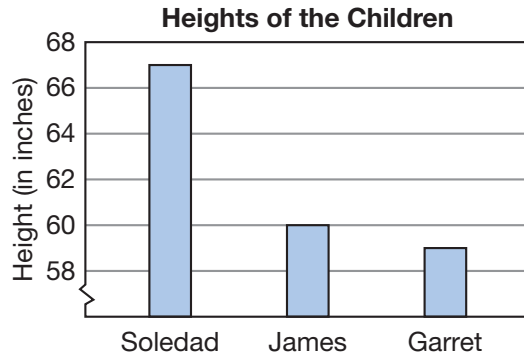


- *26. Multiple Choice** Which of these Venn diagrams illustrates the relationship between rectangles (R) and parallelograms (P)?
(45, Inv. 8)



- *27.** Write 22% as a fraction. Then reduce the fraction by dividing both terms by 2.
(71, 81)

- *28.** **Interpret** Use the graph below to answer problems **a–e**.
(50, Inv. 9)



- How many inches must Garret grow to be as tall as Soledad?
 - Which child is exactly 5 feet tall?
 - What is the average height of the three children?
 - What is the range of the heights?
 - What is the median height?
- 29.** In Alaska, Mt. McKinley is 890 meters taller than Mt. Foraker and 1198 meters taller than Mt. Blackburn. The height of Mt. Blackburn is 4996 meters. What is the height of Mt. Foraker?
(49)

- 30.** **Justify** In 1957, *Sputnik* was the first satellite launched into space. In 1976, a spacecraft named *Viking I* was the first spacecraft to land on the planet Mars. About how many years after the launch of *Sputnik* did *Viking I* land on Mars? Explain how you made your estimate.
(62)

Early Finishers
Real-World Connection

Noni surveyed 36 students in the library to find out whether they would rather learn more about the oceans or about space. Of the students surveyed, 24 students wanted to learn more about the oceans.

- Write a fraction to represent the number of students who wanted to learn more about the oceans.
- Find the greatest common factor of the numerator and denominator.
- Use the GCF to reduce the fraction.

• Properties of Geometric Solids

Power Up

facts

Power Up H

estimation

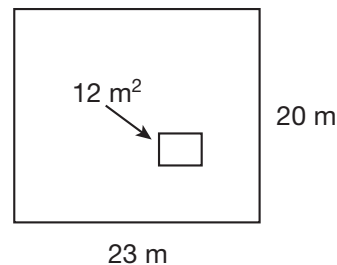
Hold your hands about one yard apart. Hold your hands about one inch apart.

mental math

- Measurement:** One mile is how many feet?
- Number Sense:** $6(20 + 3)$
- Number Sense:** $7(30 + 5)$
- Number Sense:** Reduce the fractions $\frac{2}{8}$, $\frac{4}{8}$, and $\frac{6}{8}$.
- Number Sense:** $33\frac{1}{3} + 33\frac{1}{3}$
- Probability:** To determine who will give the first speech, Alan, Bill, Christie, and Denise put their names in a hat. The teacher will draw one name. What is the probability that either Alan's or Christie's name will be drawn?
- Estimation:** Choose the more reasonable estimate for the width of a street: 25 inches or 25 feet.
- Calculation:** 50% of $\sqrt{36}$, $\times 4$, $\div 2$, $\times 6$

problem solving

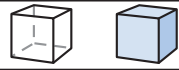
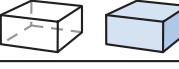




Choose an appropriate problem-solving strategy to solve this problem. Nicole reads the package of fertilizer and sees that it contains enough to cover 225 square meters. Nicole's yard has the dimensions shown at right. The rectangle in the middle of the yard represents an area where fertilizer will not be used. How many packages does Nicole need to purchase to fertilize the yard? Explain your reasoning.



New Concept

We have practiced identifying geometric shapes such as triangles, rectangles, and circles. These are “flat” shapes and are called **plane figures**. They take up a certain amount of area, but they do not take up space. Objects that take up space are things like baseballs, houses, dogs, and people.

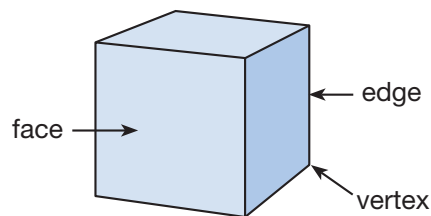
Geometric shapes that take up space are called **geometric solids**, even though real-world objects that are similar to these shapes may not be “solid.” We can make three-dimensional models of geometric solids, but they are difficult to draw on paper because paper is flat (it does not have depth). To give a sense of depth when drawing solids, we can include “hidden” edges and create optical illusions with carefully chosen angles. We draw and name some geometric solids in the table above.

Geometric Solids		
Shape		Name
		Cube
		Rectangular prism
		Pyramid
		Cylinder
		Sphere
		Cone

Math Language

Opposite faces are across from each other. Adjacent faces share an edge.

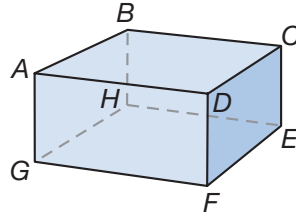
The flat surfaces of rectangular prisms and pyramids are called **faces**. A cube is a rectangular solid with 6 *congruent* faces. The opposite faces of a cube are *parallel*, and the adjacent faces of a cube are *perpendicular*. Two faces meet at an **edge**. A cube has 12 edges. Three edges meet at a *vertex*. The plural of vertex is vertices. A cube has 8 vertices.



Math Language

The bottom face of a geometric solid is called the **base**.

Rectangular prisms also have different types of line segments. Two edges that meet at the vertex form perpendicular line segments. Two edges that are parallel form parallel line segments. Two segments on opposite faces, and going in different directions, are called *skew* line segments.



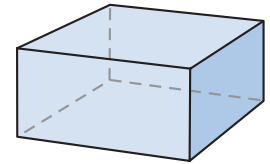
\overline{AB} and \overline{CD} are *parallel* because they are on the same plane and remain the same distance apart.

\overline{CD} and \overline{CE} are *perpendicular* because they are on the same plane and intersect at right angles.

\overline{AG} and \overline{EH} are *skew* because they are on different planes and do not intersect.

Example 1

- Name the shape at right.
- How many faces does it have?
 - This shape is a **rectangular prism**.
 - The solid has **6 faces**.



Conclude How is this solid the same as a cube? How is it different?

Example 2

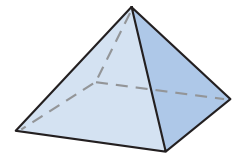
What is the shape of a basketball?

A basketball is not a circle. A circle is a “flat” shape (a plane figure), but a basketball takes up space. A basketball is a **sphere**.

Example 3

Name this shape and identify its congruent parts.

The shape is a **pyramid**. It has 4 **congruent triangular faces**, and, in this case, 1 square base.

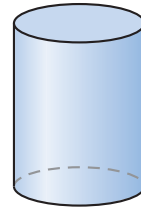


Discuss Are any faces of a pyramid parallel? Are any faces perpendicular to the base? Explain why or why not.

Example 4

Name this shape and identify its congruent parts.

The shape is a **cylinder**. The two **circular surfaces** of the cylinder are congruent, parallel bases.



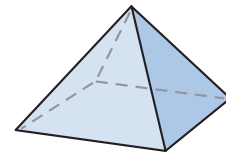
Lesson Practice

Connect Name the geometric shape of each of these real-world objects:

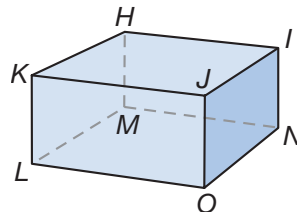
- a. brick
- b. soup can
- c. ice cream cone
- d. shoe box

Refer to the pyramid to answer problems e–h.

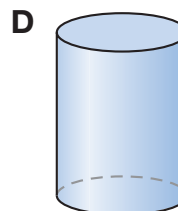
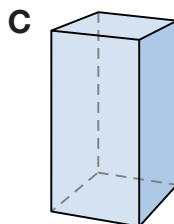
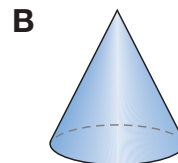
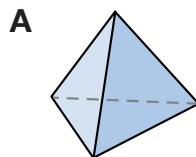
- e. The pyramid has how many triangular faces?
- f. The pyramid has how many rectangular faces?
- g. The pyramid has how many edges?
- h. The pyramid has how many vertices?



Refer to the rectangular solid to answer problems i–k.

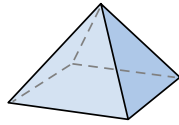


- i. Name a pair of parallel line segments.
- j. Name a pair of perpendicular line segments.
- k. Name a pair of skew line segments.
- l. **Multiple Choice** Which figure below has faces that are perpendicular to its base?

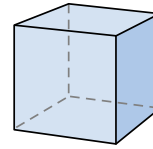


m. **Multiple Choice** Which figure below has six congruent faces?

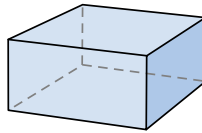
A



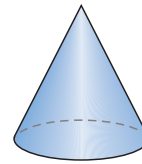
B



C




D



Written Practice

Distributed and Integrated

1.  **Explain** Alycia left for school at a quarter to eight in the morning and arrived home $7\frac{1}{2}$ hours later. What time was it when Alycia arrived home? How do you know?

*2. (64, 70) D'Mitra has 5 coins in her pocket that total 47¢. How many dimes are in her pocket?

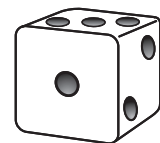
3. **Represent** (52) Use digits to write the number twenty-three million, two hundred eighty-seven thousand, four hundred twenty.

4. (Inv. 3) a. What number is $\frac{1}{3}$ of 24?
b. What number is $\frac{2}{3}$ of 24?

5. **List** (80) Write all the prime numbers between 10 and 20.

*6. (82) a. What is the greatest common factor (GCF) of 4 and 8?
b. Use the GCF of 4 and 8 to reduce $\frac{4}{8}$.

*7. (83) a. Name this shape.
b. How many faces does it have?



*8. **Multiple Choice** (83) Which geometric figure best describes the shape of the earth?

A circle

B cylinder

C sphere

D plane

9. Write a decimal number equal to the mixed number $1\frac{7}{10}$.

*10. **Multiple Choice** Which word names the distance across a circle?
 (53) **A** center **B** circumference **C** radius **D** diameter

11. $3.62 + 4.5$
(73)

12. $3.704 - 2.918$
(73)

*13. $16^2 + \sqrt{16}$
(78)

14. $\$6.25 \times 4$
(17)

15. $6w = \$14.58$
(26)

*16. **Analyze** Write a fraction equal to $\frac{1}{3}$ that has a denominator of 12. Then write a fraction equal to $\frac{3}{4}$ that has a denominator of 12. What is the sum of the two fractions you wrote?
(79)

*17. Reduce: $\frac{6}{8}$
(81)

*18. $\frac{3}{4} = \frac{\square}{12}$
(79)

19. $4\frac{1}{6}$
 $+ 2\frac{1}{6}$

(43, 71)

20. $3\frac{3}{4}$
 $+ 1\frac{1}{4}$

(59)

21. 5
 $- 1\frac{1}{4}$

(63)

22. Compare: $0.1 \bigcirc 0.01$
(69)

23. Since $3 \times \frac{1}{2}$ is the same as $\frac{1}{2} + \frac{1}{2} + \frac{1}{2}$, what mixed number is the same as $3 \times \frac{1}{2}$?
(75)

24. Use the information and the table below to answer parts **a** and **b**.
(11, 23)

Mr. and Mrs. Minick took their children, Madison and Douglas, to a movie. Ticket prices are shown in the table.

Movie Ticket Prices

Age	Price
Adults	\$10.00
Ages 9–12	\$8.50
Under 9	\$6.50

a. Madison is 12 years old and Douglas is 8 years old. What is the total cost of all four tickets?

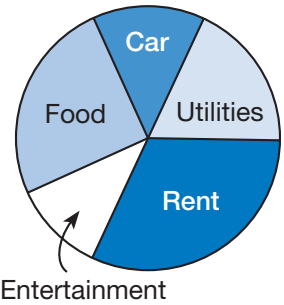
b. Before 5 p.m., all tickets are \$6.50. How much money would the Minicks save by going to the movie before 5 p.m. instead of after 5 p.m.?

*25. **Multiple Choice** Which of these figures is an illustration of an object that “takes up space”?
(83)



26. **Estimate** The area of a room that is 14 ft 2 in. long and 10 ft 3 in. wide is about how many square feet?

*27. **Interpret** The pie chart at right shows how a family's monthly expenses are divided.

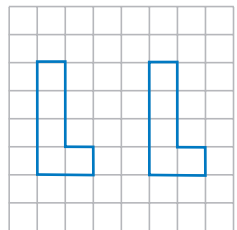


- a. Which expense consumes about one third of the budget?
- b. About what fraction of the budget does food consume?

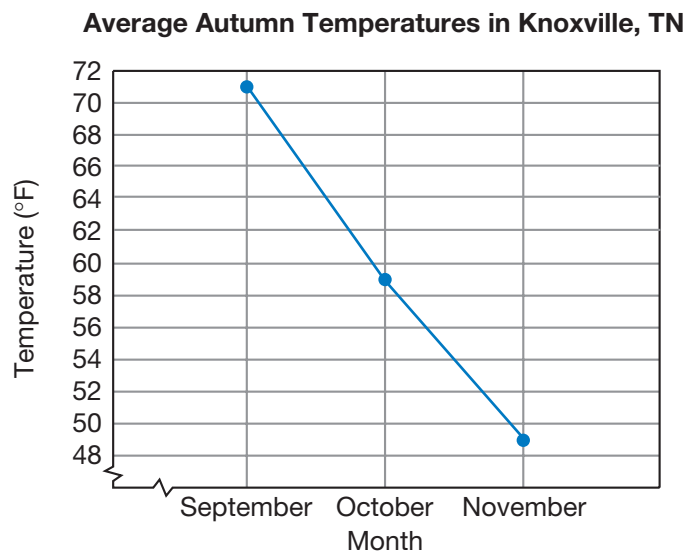
28. What is the perimeter of a rhombus with sides 2.4 centimeters long?

*29. **Multiple Choice** Which transformation moves the figure from one position to the other?

- A translation
- B rotation
- C reflection
- D flip



30. **Interpret** The line graph shows the average monthly temperatures during autumn in Knoxville, Tennessee. Use the graph to answer the questions that follow.



- a. What number of degrees represents the range of the temperatures?
- b. How many degrees warmer or cooler is the October temperature than the September temperature?
- c. How many degrees warmer or cooler is the October temperature than the November temperature?

• Mean, Median, Mode, and Range

Power Up

facts

Power Up H

mental math

- Number Sense:** $9(30 + 2)$
- Number Sense:** $8(30 + 4)$
- Number Sense:** Reduce the fractions $\frac{2}{6}$, $\frac{3}{6}$, and $\frac{4}{6}$.
- Measurement:** An adult horse can weigh about half a ton. Half a ton equals how many pounds?
- Percent:** 10% of \$500
- Powers/Roots:** 7^2
- Probability:** Manny plans to flip a coin 50 times and record the results. Is it certain, likely, unlikely, or impossible that *all* the coin flips will be tails?
- Calculation:** $\frac{1}{3}$ of 60, $+ 1$, $\div 3$, $\times 5$, $+ 1$, $\div 4$

problem solving

Choose an appropriate problem-solving strategy to solve this problem. A **permutation** is an arrangement of numbers or objects in a particular order. For example, if we take the combination (1, 2, 3) from the set of counting numbers, we can form six permutations. Four of the permutations are (1, 2, 3), (1, 3, 2), (2, 1, 3), and (2, 3, 1). What are the remaining two permutations for these three numbers?

New Concept

In Lesson 50, we found the *average* of a set of numbers, and in Investigation 5, we learned about the median, mode, and range of a set of numbers. In this lesson, we will review these terms.

Math Language

Mean, median, mode, and range are different ways to summarize a set of data with a number.

The average is also called the **mean**. To find a mean, we add and then divide. For example, suppose we wanted to know the mean number of letters in the following names: Andrei, Raj, Althea, Nina, Bedros, Ann, and Yolanda.

Name	Andrei	Raj	Althea	Nina	Bedros	Ann	Yolanda
Number of Letters	6	3	6	4	6	3	7

We first add the seven numbers 6, 3, 6, 4, 6, 3, and 7. Then we divide the resulting sum by 7.

$$\text{Add: } 6 + 3 + 6 + 4 + 6 + 3 + 7 = 35$$

$$\text{Divide: } 35 \div 7 = 5$$

The mean number of letters is 5. Notice that no name contains 5 letters. The mean of a set of numbers does not have to be one of the numbers. In fact, the mean of a set of whole numbers can even be a mixed number.

Example 1

Find the mean of this data set: 2, 7, 3, 4, 3

We divide the sum of the data points (19) by the number of data points (5). We write the remainder as a fraction and find that the mean of the data set is $3\frac{4}{5}$.

$$\begin{array}{r} 3\frac{4}{5} \\ 5 \overline{)19} \\ \underline{15} \\ 4 \end{array}$$

Recall that the middle number of a data set, when the data are arranged in numerical order, is called the **median**.

Example 2

Kayla tracked the number of days it rained each month during the school year and recorded the totals in a table.

Month	S	O	N	D	J	F	M	A	M
Number of Rainy Days	3	5	8	2	5	7	7	6	1

Find the median number of rain days per school month.

We first put the data in numerical order: 1, 2, 3, 5, 5, 6, 7, 7, 8. The middle object in a row of objects has the same number of objects on its left as it has on its right.

$$\begin{array}{ccccccc} 1 & 2 & 3 & 5 & \textcircled{5} & 6 & 7 & 7 & 8 \\ \underbrace{\hspace{4em}} & & & & & \underbrace{\hspace{4em}} & & & \\ 4 \text{ objects} & & & & & 4 \text{ objects} & & & \\ \text{to the left} & & & & & \text{to the right} & & & \end{array}$$

We see that the median is **5 days of rain**.

Thinking Skill

Discuss

To find a median, why is it helpful to write the data in order from greatest to least or from least to greatest?

If a data set has an even number of data points, there are two middle numbers. In these cases, the median is the average of the two middle numbers.

Example 3

Jordan recorded the number of inches of snow that fell during the first eight weeks of winter.

Week	1	2	3	4	5	6	7	8
Number of Inches of Snow	2	5	1	6	9	8	3	10

Find the median number of inches of snow that fell during these weeks.

We arrange the numbers in numerical order to get the list 1, 2, 3, 5, 6, 8, 9, 10. The two middle numbers are 5 and 6. The median is the average of 5 and 6. We add 5 and 6 and then divide the resulting sum by 2.

$$\begin{array}{cccccccc}
 1 & 2 & 3 & \textcircled{5} & \textcircled{6} & 8 & 9 & 10 \\
 & & & \underbrace{\hspace{1.5cm}} & & & & \\
 \rightarrow & \frac{5 + 6}{2} = \frac{11}{2} = 5\frac{1}{2}
 \end{array}$$

The median is $5\frac{1}{2}$ inches of snow.

Returning to our list of names at the beginning of this lesson, we find that the most common number of letters in a name is 6. There are three names with 6 letters: Andrei, Althea, and Bedros. If some data points occur more than once, remember that the one that occurs most often is called the **mode**. There can be more than one mode for a data set.

Example 4

The bank manager recorded the number of new savings accounts during the first nine business days of the month.

Day	1	2	3	4	5	6	7	8	9
Number of New Accounts	3	5	8	2	5	7	7	6	1

Find the mode of this set of data.

The numbers 5 and 7 both appear twice. No other numbers appear more than once. So there are two modes: **5** and **7**.

Mean, median, and mode are different ways to describe the *center* of a data set. They are called **measures of central tendency**. We might also be interested in the **spread** of a data set. Spread refers to how the data are stretched out. The simplest measure of spread is the **range**. Recall that the range is the difference between the largest and smallest data points. For example, in Example 4, the largest number of new accounts is 8 and the smallest is 1. So the range for the data set is 7 because $8 - 1 = 7$.

Lesson Practice

Find the mean, median, mode, and range of each data set in problems **a–c**.

- a. 3, 7, 9, 9, 4
- b. 16, 2, 5, 7, 11, 13
- c. 3, 10, 2, 10, 10, 1, 3, 10
- d. **Analyze** Find the mean, median, mode, and range for the ages of the students in this table:

Name	Andrei	Raj	Althea	Mary	Bedros	Ann	Yolanda
Age	13	10	10	11	11	10	11

Written Practice

Distributed and Integrated

1. ⁽²¹⁾ Three families are equally sharing the \$2475 cost of renting a large home on a lake. What is each family's share of the cost?
2. ^(Inv. 3, 37) **Analyze** Draw a circle and shade $\frac{1}{3}$ of it. What percent of the circle is shaded?
3. ⁽⁷⁷⁾ A pair of Jay's dress shoes weighs about a kilogram. A kilogram is how many grams?
4. ⁽⁶²⁾ **Estimate** Write the product of 732 and 480 by rounding the numbers to the nearest hundred before multiplying.
- *5. ^(28, 32) **Multiple Choice** At which of these times do the hands of a clock form an acute angle?
A 3:00 **B** 6:15 **C** 9:00 **D** 12:10

6. Arrange these decimal numbers in order from least to greatest:

(69)

0.1, 0.01, 1.0, 1.01

*7. a. Find the common factors of 8 and 12.

(82)

b. Use the GCF of 8 and 12 to reduce $\frac{8}{12}$.

8. a. What number is $\frac{1}{4}$ of 80?

(Inv. 2)

b. What number is $\frac{3}{4}$ of 80?

*9. $\frac{1}{2} \times \square = \frac{3}{6}$

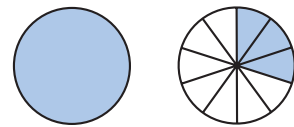
(79)

*10. Reduce: $\frac{4}{6}$

(81)

11. **Connect** Name the total number of shaded circles as a mixed number and as a decimal number.

(71)



12. $9.9 + 6.14 + 7.5 + 8.31$

(73)

13. $\$10 - 59\text{¢}$

(70)

14. $30 \overline{)672}$

(54)

*15. $5 \times 68\text{¢} = \$______$

(70)

*16. $\$3.40 \div 5$

(26)

17. $10 - 3\frac{1}{3}$

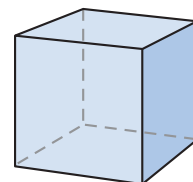
(63)

18. $\frac{3}{4} \times \frac{5}{4}$

(76)

*19. **Classify** Describe this geometric solid using the words *perpendicular* and *parallel*.

(83)



20. **Multiple Choice** In rectangle $MNOP$, which segment is parallel to \overline{MN} ?

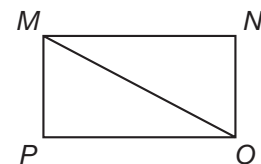
(31, 61)

A \overline{MP}

B \overline{PO}

C \overline{NO}

D \overline{MO}



*21. **Multiple Choice** Which angle in this figure appears to be a right angle?

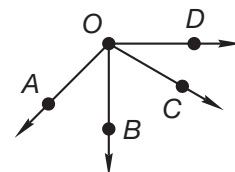
(31, 61)

A $\angle AOB$

B $\angle BOC$

C $\angle BOD$

D $\angle AOD$



***22. Analyze** Use the grocery receipt to answer parts **a–e**.

(73, 84)

- a. How much money was spent on eggs, juice, and cereal?
- b. What was the average (mean) price of the eight items?
- c. What is the median price of the eight items?
- d. What is the mode of the prices?
- e. What is the range of the eight prices?

Milk	1.94
Milk	1.94
Milk	1.94
Milk	1.94
Apple juice . .	1.38
Apple juice . .	1.38
Eggs	3.10
Cereal	3.98
<hr/>	
TOTAL	17.60

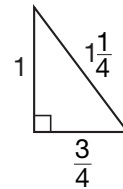
***23. Conclude** The first three *triangular numbers* are 1, 3, and 6, as illustrated by the number of dots that make up each of the figures below. Find the next triangular number by drawing the next figure in the pattern.

(Inv. 4)



24. Find the perimeter of this right triangle. Units are in inches.

(53, 59)



25. Analyze Write 90% as a fraction. Then reduce the fraction by dividing both terms by 10. What decimal number does the fraction equal?

(71, 81)

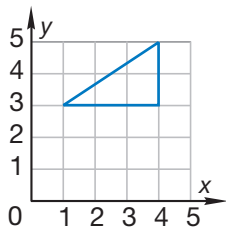
26. On the Fahrenheit scale, how many degrees are between the temperature at which water freezes and the temperature at which water boils?

(27)

27. Estimate A full moon, crescent moon, and new moon are examples of phases of our moon. It takes the moon about $29\frac{1}{2}$ days to complete one cycle of phases. Estimate the number of cycles of phases the moon completes in one year, and explain why your estimate is reasonable. (*Hint:* One year is about 365 days.)

(62)

- *28.** Copy this grid and triangle onto your paper. Then draw a translation of the triangle shifted down 2 units.
(Inv. 8)



- *29.** The maximum depth in feet of three natural lakes is shown in the table.
(Inv. 5) Display the data in a vertical bar graph. Remember to include a key.

Lake	Continent	Maximum Depth (feet)
Victoria	Africa	270
Nipigon	North America	540
Reindeer	North America	720

- 30.** The wall-to-wall carpeting in a family room needs to be replaced. The room measures 16 feet long by 12 feet wide by 8 feet high. What area represents the amount of carpeting that will be replaced?
(72)

Early Finishers

Real-World Connection

For five days in a row, Roberto recorded the number of visits that were made to his classroom's website. The least number of visits in a day was 7. The greatest number of visits in a day was 14. The mode of the group of data is 9. The median is 9, and the mean is 10.

- Use this information to find the number of visits made to the classroom's website during each of the five days.
- What is the range of the data set?

• Units of Capacity

Power Up

facts

Power Up H

mental math

- Measurement:** $3 \times (2 \text{ pounds } 4 \text{ ounces})$
- Measurement:** $3 \times (4 \text{ pounds } 6 \text{ ounces})$
- Measurement:** One mile equals how many feet?
- Measurement:** Half a pound equals how many ounces?
- Number Sense:** Reduce the fractions $\frac{2}{10}$, $\frac{4}{10}$, $\frac{6}{10}$, and $\frac{8}{10}$.
- Number Sense:** $33\frac{1}{3} + 66\frac{2}{3}$
- Probability:** The sides of a number cube are labeled 1 through 6. If the cube is rolled once, what is the probability that the number will be a 5 or 6?
- Calculation:** $\frac{1}{2}$ of 5, $\times 2$, $\times 5$, $\times 4$

problem solving

Choose an appropriate problem-solving strategy to solve this problem. Two cups equal a pint. Two pints equal a quart. Two quarts equal a half gallon. Two half gallons equal a gallon. One pint and one quart is a total of how many cups?

New Concept

Thinking Skill

Analyze

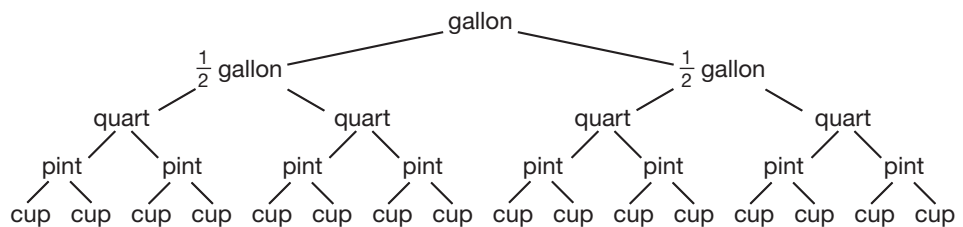
A liter is a little more than a quart. About how many liters are equal to 1 gallon?

When we buy milk, water, or fruit juice at the store, we are buying a quantity of liquid. In the U.S. Customary System, liquid quantities are measured in **ounces** (oz), pints (pt), quarts (qt), and gallons (gal). In the metric system, liquid quantities are

measured in **liters** (L) and milliliters (mL). Here we show some common containers of liquids:



These cartons and bottles have **capacity**. A container's capacity refers to the amount of liquid it can hold. Many containers in the U.S. Customary System are related by a factor of 2. One gallon is 2 half gallons. A half gallon is 2 quarts. A quart is 2 pints. A pint is 2 cups. We show these relationships in the following diagram:



The table below shows some common units of liquid measure. The table also shows equivalencies between the units.

Equivalency Table for Units of Liquid Measure

U.S. Customary System	Metric System
16 oz = 1 pt 2 pt = 1 qt 4 qt = 1 gal	1000 mL = 1 L

A liter is about 2 ounces more than a quart.

Thinking Skill

Discuss

The U.S. Customary System also includes units of measure for smaller amounts. For example, 1 tablespoon = $\frac{1}{2}$ ounce and 1 teaspoon = $\frac{1}{6}$ ounce. How many teaspoons are equal to 1 tablespoon? Explain why.

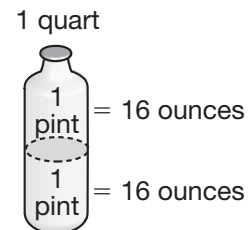
Example 1

Math Language

The word *ounce* is used to describe a weight as well as an amount of liquid. The liquid measurement *ounce* is often called a **fluid ounce**. Although *ounce* has two meanings, a fluid ounce of water does weigh about 1 ounce.

One quart of juice is how many ounces of juice?

The table tells us that a quart is 2 pints and that each pint is 16 ounces. Since 2 times 16 is 32, 1 quart is the same as **32 ounces**.



Example 2

A half gallon of milk is how many quarts of milk?

A whole gallon is equal to 4 quarts. A half gallon is equal to half as many quarts. A half gallon equals **2 quarts**.

Example 3

In the table below, we see the number of pints in 1 gallon, 2 gallons, and 3 gallons.

Gallons	1	2	3	4	5
Pints	8	16	24	?	?

How many pints are in 4 gallons? 5 gallons? $5\frac{1}{2}$ gallons?

One gallon equals 8 pints, so 4 gallons equals **32 pints** and 5 gallons equals **40 pints**. Since half a gallon is half of 8 pints, we find that $5\frac{1}{2}$ gallons equals **44 pints**.

Example 4

A half liter equals 500 mL. How many milliliters are equal to $3\frac{1}{2}$ liters?

Each liter is 1000 mL, so $3\frac{1}{2}$ liters is $3000 \text{ mL} + 500 \text{ mL} = \mathbf{3500 \text{ mL}}$.

Lesson Practice

- One fourth of a dollar is a quarter. What is the name for one fourth of a gallon?
- How many pints equal 1 gallon?
- How many milliliters equal 2 liters?
- A cup is one half of a pint. A cup is how many ounces?

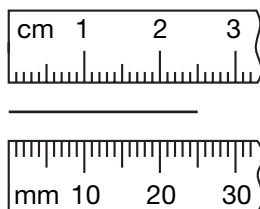
Written Practice

Distributed and Integrated

- Represent** Draw a rectangle. Shade all but two fifths of it. What percent of the rectangle is shaded?
(71)
- Analyze** Write a three-digit prime number using the digits 4, 1, and 0 once each.
(80)

3. Find the length of this segment in centimeters and in millimeters:

(66)



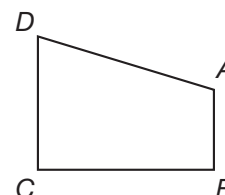
4. **Analyze** Tisha counted her heartbeats. Her heart beat 20 times in 15 seconds. At that rate, how many times would it beat in 1 minute?

(49)

- *5. **Multiple Choice** In this quadrilateral, which segment appears to be perpendicular to \overline{AB} ?

(31, 61)

- A \overline{BC} B \overline{CD} C \overline{DA} D \overline{DC}



- *6. a. Find the common factors of 6 and 9.

(82)

- b. Use the GCF of 6 and 9 to reduce $\frac{6}{9}$.

7. a. What number is $\frac{1}{5}$ of 60?

(Inv. 3)

- b. What number is $\frac{2}{5}$ of 60?

- *8. \overline{AB} is $1\frac{1}{4}$ inches. \overline{BC} is $2\frac{1}{4}$ inches. Find \overline{AC} .

(61, 81)



9. Arrange these numbers in order from least to greatest:

(69)

0.1, 0, 0.01, 1.0

- *10. Four quarts of water is how many pints of water?

(85)

- *11. Three liters equals how many milliliters?

(85)

- *12. Divide 100 by 6 and write the quotient as a mixed number. Then rewrite the quotient by reducing the fraction part of the mixed number.

(58, 81)

13. $\$17.56 + \$12 + 95\text{¢}$

(70)

14. $4.324 - 1.91$

(73)

15. $\begin{array}{r} 396 \\ \times 405 \\ \hline \end{array}$

16. $\$1.25 \times 20$

17. $9 \overline{)3605}$

18. $\$2.50 \div 10$

19. Reduce: $\frac{15}{20}$

20. $3 - \left(2\frac{2}{3} - 1\right)$

- *21. **Analyze** Write a fraction equal to $\frac{3}{5}$ that has a denominator of 10. Then write a fraction equal to $\frac{1}{2}$ that has a denominator of 10. What is the sum of the two fractions you wrote?

22. When five and twelve hundredths is added to six and fifteen hundredths, what is the sum? Write your answer as a decimal number.

23. Since $\frac{1}{4} + \frac{1}{4} + \frac{1}{4} = \frac{3}{4}$, how many $\frac{1}{4}$ s are in $\frac{3}{4}$?

24. Use this information to answer parts **a** and **b**:

Stan is 6 inches taller than Roberta. Roberta is 4 inches shorter than Genaro. Genaro is 5 feet 3 inches tall.

a. How tall is Roberta?

b. How tall is Stan?


- *25. The first term of a certain sequence is 4. Each term that follows is found by doubling the number before it. Write the first four terms of the sequence.


26. If you toss a coin 50 times, about how many times would you expect it to land heads up?

- *27. The scores of Adia's first seven games are listed below. Use the information below to answer parts **a–c**.

90, 85, 80, 90, 95, 90, 100

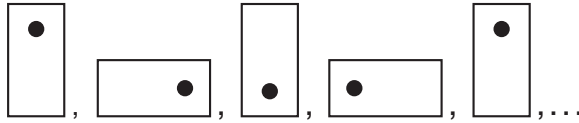
a. What is the range of the scores?

b.  **Justify** What is the mode of the scores? Why?


c.  **Justify** What is the median score? Why?

28. Coretta has 2 more letters in her last name than Justin has in his last name. Maya has 3 fewer letters in her last name than Justin has. Maya has 5 letters in her last name. How many letters does Coretta have in her last name?

- *29. Refer to the sequence below to answer parts **a** and **b**.
(Inv. 4, 88)



- a. What is the next term of the sequence? Draw a figure on your paper.
- b. **Multiple Choice** Which transformation describes the change from term to term in this sequence?
A translation B rotation C reflection D flip

30.  **Estimate** At Central Middle School, 154 students are enrolled in grade 6, 147 students are enrolled in grade 7, and 133 students are enrolled in grade 8. What is a reasonable estimate of the number of students enrolled in grades 6–8?

Early Finishers
Real-World Connection

Suri is planning a party for 80 people. She would like to serve punch to every guest.

- a. If Suri serves each guest 1 cup of punch, how many gallons of punch will she need?
- b. If Suri serves each guest $1\frac{1}{2}$ cups of punch, how many gallons of punch will she need?

• Multiplying Fractions and Whole Numbers

Power Up

facts

mental math

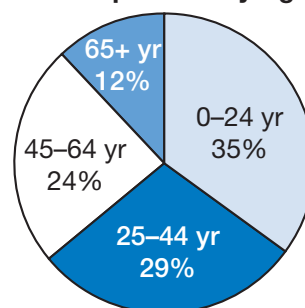
Power Up H

- Money:** $3 \times (\$6 \text{ and } 25\text{¢})$
- Money:** $5 \times (\$3 \text{ and } 25\text{¢})$
- Money:** One dollar is how many quarters?
- Measurement:** How many quarts are in a gallon?
- Powers/Roots:** 8^2
- Probability:** The sides of a number cube are labeled 1 through 6. If the cube is rolled once, what is the probability of rolling a number less than 5?
- Estimation:** Choose the more reasonable estimate for the mass of a hamster: 90 grams or 90 kilograms.
- Calculation:** $\frac{1}{3}$ of 90, $+ 3$, $\div 3$, $\times 9$

problem solving

Choose an appropriate problem-solving strategy to solve this problem. The circle graph at right is based on year 2004 estimates published by the Census Bureau. The graph shows the percentage of United States residents that belong to each of four age groups. Every 1 percent of the population is equal to nearly 3 million residents. Find the approximate number of U.S. residents who are 65 years of age or older.

U.S. Population by Age



New Concept

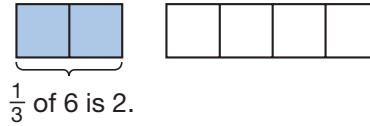
We have found a fraction of a whole number by dividing the whole number by the denominator of the fraction.

$$\frac{1}{3} \text{ of } 6 \text{ is } 2. \quad (6 \div 3 = 2)$$

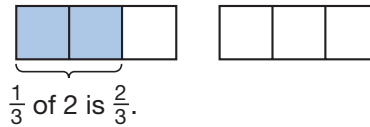
Reading Math

When we multiply with fractions, the answer is stated in terms of the whole.

The model below illustrates that $\frac{1}{3}$ of 6 rectangles is 2 rectangles.



How can we model $\frac{1}{3}$ of 2? If we divide two whole rectangles into three parts each, then there are 6 parts in all, and $\frac{1}{3}$ of 6 parts is 2 parts. We see that 2 parts is $\frac{2}{3}$ of a whole rectangle.



An arithmetic method for finding $\frac{1}{3}$ of 2 is to multiply.

What number is $\frac{1}{3}$ of 2?

$$\begin{array}{c} \frac{1}{3} \text{ of } 2 \\ \downarrow \downarrow \downarrow \\ \frac{1}{3} \times \frac{2}{1} \end{array}$$

Notice that we wrote the whole number 2 as a fraction: $\frac{2}{1}$. Since 2 divided by 1 is 2, the fraction $\frac{2}{1}$ equals 2. Writing the whole number as a fraction gives us a numerator and a denominator to multiply. The product is $\frac{2}{3}$.

$$\frac{1}{3} \times \frac{2}{1} = \frac{2}{3}$$

Now we check for reasonableness. We know that $\frac{1}{2}$ of 2 is 1. Since $\frac{1}{3}$ is less than $\frac{1}{2}$, $\frac{1}{3}$ of 2 must be less than 1, and $\frac{2}{3}$ is less than 1.

Here is another way to check our answer. Recall the Commutative Property of Multiplication. This property tells us that changing the order of factors does not affect the product. So another way to approach this problem is to switch the positions of $\frac{1}{3}$ and 2.

$$\begin{array}{c} \frac{1}{3} \times 2 \\ \swarrow \searrow \\ 2 \times \frac{1}{3} \end{array} \quad \begin{array}{l} \text{We may reverse the order of} \\ \text{factors when we multiply.} \end{array}$$

Since $2 \times \frac{1}{3}$ means $\frac{1}{3} + \frac{1}{3}$, we again find that the product is $\frac{2}{3}$.

Example

What number is $\frac{2}{3}$ of 4?

We know that $\frac{2}{3}$ of 4 is greater than 2 because $\frac{1}{2}$ of 4 is 2, and $\frac{2}{3}$ is greater than $\frac{1}{2}$. We also know that $\frac{2}{3}$ of 4 is less than 4. We multiply to find the answer.

$$\begin{array}{c} \frac{2}{3} \text{ of } 4 \\ \downarrow \downarrow \downarrow \\ \frac{2}{3} \times \frac{4}{1} = \frac{8}{3} = 2\frac{2}{3} \end{array}$$

We converted the improper fraction to a mixed number. Since $2\frac{2}{3}$ is greater than 2 but less than 4, the answer is reasonable. We can check the answer by reversing the order of factors.

$$4 \times \frac{2}{3} \text{ means } \frac{2}{3} + \frac{2}{3} + \frac{2}{3} + \frac{2}{3}$$

Again we get $\frac{8}{3}$, which equals $2\frac{2}{3}$.

Lesson Practice

Multiply. Simplify answers when possible. Reverse the order of factors to check your answer.

a. $\frac{1}{3} \times 4$

b. $\frac{3}{5} \times 2$

c. $\frac{2}{3} \times 2$

d. What number is $\frac{1}{5}$ of 4?

e. What number is $\frac{1}{6}$ of 5?

f. What number is $\frac{2}{3}$ of 5?

g. **Model** Sketch rectangles to model $\frac{1}{3}$ of 4. Begin by drawing four rectangles and then divide each rectangle into thirds. Then find $\frac{1}{3}$ of the total number of parts.

Written Practice

Distributed and Integrated

*1. **Represent** Draw a pair of horizontal parallel segments. Make the lower segment longer than the upper segment. Make a quadrilateral by connecting the endpoints.
(31, 32)

2. **Estimate** Find the difference between 6970 and 3047 by rounding the numbers to the nearest thousand and then subtracting.
(62)

3. **Represent** Write the following sentence using digits and symbols:

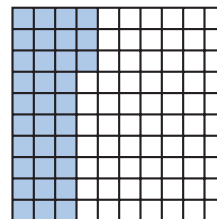
The sum of six and four is ten.

- *4. A 2-liter bottle of water contains how many milliliters of liquid?

(85)

5. Name the shaded portion of this square as a fraction, as a decimal number, and as a percent:

(71)



- *6. a. What number is $\frac{1}{3}$ of 120?

(86)

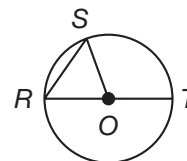
- b. What number is $\frac{2}{3}$ of 120?

7. **Multiple Choice** Which segment names a diameter of this circle?

(53, 61)

- A \overline{RS}
C \overline{OS}

- B \overline{RT}
D \overline{OT}



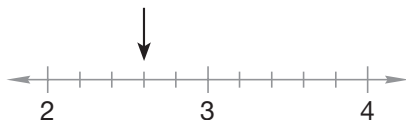
8. **List** Write these fractions in order from least to greatest:

(59, 75)

$$\frac{9}{18}, \frac{8}{7}, \frac{7}{16}, \frac{6}{6}, \frac{5}{8}$$

9. **Connect** To what mixed number is the arrow pointing?

(38)



Multiply to find each product in problems 10 and 11. Then reverse the order of factors to check your answers.

*10. $\frac{2}{3} \times 2$

(86)

*11. $\frac{3}{4}$ of 4

(86)

12. $3 - \left(2\frac{3}{5} - 1\frac{1}{5}\right)$

(41, 63)

13. $4.7 + 3.63 + 2.0$

(73)

$$\begin{array}{r} 14. \quad 301.4 \\ (73) \quad - 143.5 \\ \hline \end{array}$$

$$\begin{array}{r} 15. \quad 476 \\ (56) \quad \times 890 \\ \hline \end{array}$$

$$16. \quad 4 \overline{)348}$$

(26)

$$17. \quad 40 \overline{)3480}$$

(54)

$$18. \quad \$42.36 \div 6$$

(34)

$$19. \quad 22^2$$

(78)

- *20. a. What are the common factors of 60 and 100?
(82)

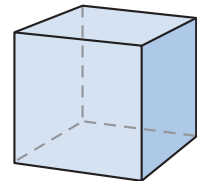
b. Use the GCF of 60 and 100 to reduce $\frac{60}{100}$.

- *21. Write a fraction equal to $\frac{3}{4}$ that has a denominator of 12. Then write a fraction equal to $\frac{2}{3}$ that has a denominator of 12. Subtract the second fraction from the first fraction.
(79)

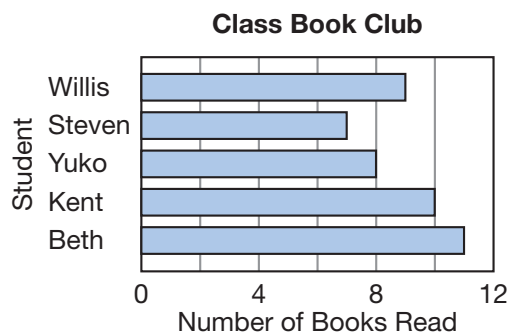
22. Since $\frac{3}{4} + \frac{3}{4} + \frac{3}{4} = \frac{9}{4}$, how many $\frac{3}{4}$ s are in $\frac{9}{4}$?
(Inv. 2)

- *23. a. What is the name of this solid?
(83)

b. How many vertices does it have?




- *24. **Interpret** Use the graph below to answer parts a–c.
(Inv. 7, 84)



- a. How many more books must Steven read to reach the goal of 12 books?
- b. Each book must have 180 pages or more. Kent has read at least how many pages so far?
- c. What is the median number of books read by the five students?
- *25. What is the probability of rolling a number less than five with one toss of a standard number cube? Write the probability as a reduced fraction.
(57, 81)

*26. A quart is called a quart because it is a quarter of a gallon. What percent of a gallon is a quart?
(71, 85)

*27. Compare: 1 quart ○ 1 liter
(85)

*28.  **Explain** For exercise, Jia walks $1\frac{1}{2}$ miles each morning and $2\frac{1}{2}$ miles each evening. At that rate, how many days will it take Jia to walk 100 miles? Explain how you found your answer.
(49, 75)

*29. Refer to the following sequence to answer parts **a** and **b**.
(Inv. 8)

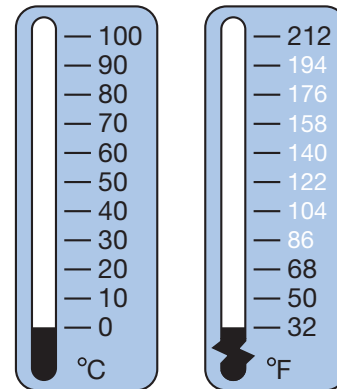


a. What is the next term in the sequence? Draw the answer on your paper.

b. **Multiple Choice** Which transformation describes the change from term to term?

- A** translation **B** rotation **C** reflection **D** slide

*30. A difference of 100° on the Celsius scale is a difference of 180° on the Fahrenheit scale. A 10° change on the Celsius scale is an 18° change on the Fahrenheit scale. Copy this thermometer on your paper, and label the remaining tick marks on the Fahrenheit scale.
(27, Inv. 4)



Early Finishers

Real-World Connection

Cherise is baking a cake. She wants to make a smaller cake that is $\frac{2}{3}$ the size of the original recipe. If 3 cups of flour and 2 cups of milk are needed to make the cake in the recipe, how much flour and milk will she need to make the smaller cake?

• Using Manipulatives and Sketches to Divide Fractions

Power Up

facts

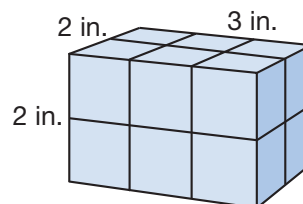
Power Up H

mental math

- Measurement:** $3 \times (2 \text{ ft } 4 \text{ in.})$
- Measurement:** $4 \times (3 \text{ ft } 4 \text{ in.})$
- Powers/Roots:** 9^2
- Estimation:** Choose the more reasonable estimate for the amount of water in a drinking glass: 300 milliliters or 3 liters.
- Number Sense:** Reduce the fractions $\frac{2}{10}$, $\frac{2}{12}$, $\frac{2}{14}$, and $\frac{2}{16}$.
- Percent:** Chad has hiked 25% of the 4-mile trail. How many miles does he have left to hike?
- Probability:** Nine of the ten boxes contain a hidden prize. Latrisha will choose one box to open. Is it certain, likely, unlikely, or impossible that she will get a prize?
- Calculation:** $\frac{1}{4}$ of 400, $\div 2$, $- 5$, $\div 5$, $\times 4$, $\div 6$

problem solving

Choose an appropriate problem-solving strategy to solve this problem. Some 1-inch cubes were used to build this rectangular solid. How many 1-inch cubes were used?



New Concept

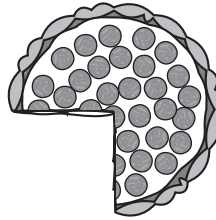
In this lesson, we will use fraction manipulatives and make sketches to help us divide fractions. First, let us think about what dividing fractions means.

Reading Math

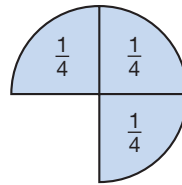
The division $3\overline{)4}$ is the same as $4 \div 3$.
 The division $\frac{3}{4} \div \frac{1}{8}$ is the same as $\frac{1}{8}\overline{)3}$.

$$\frac{3}{4} \div \frac{1}{8}$$

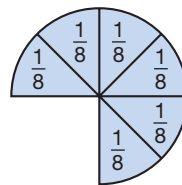
The expression above means, "How many one eighths are in three fourths?"; that is, how many one-eighth slices of pizza are in three fourths of a pizza?



Using manipulatives, we place three fourths on our desk.



If we cover the three fourths with eighths, we can see that there are 6 one eighths in three fourths.



$$\frac{3}{4} \div \frac{1}{8} = 6$$

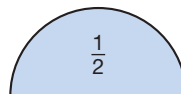
Example 1

Model How many one eighths are in one half?

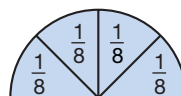
This is a division question. It could also be written

$$\frac{1}{2} \div \frac{1}{8}$$

Using our fraction manipulatives, we place one half on our desk.



To find how many one eighths are in one half, we cover the half with eighths and then count the eighths.

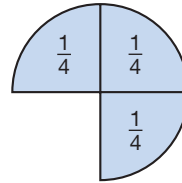


The answer is **4**. There are 4 one eighths in one half.

Example 2

Model Divide: $\frac{3}{4} \div \frac{1}{4}$

This problem means, “How many one fourths are in three fourths?” We make three fourths of a circle using fourths. Then we count the fourths.



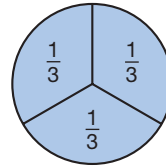
There are 3 one fourths in three fourths.

$$\frac{3}{4} \div \frac{1}{4} = 3$$

Example 3

Model Divide: $1 \div \frac{1}{3}$

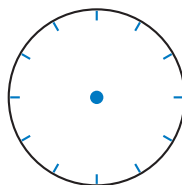
This problem means, “How many one thirds are in one?” Using our manipulatives, we want to find the number of one-third pieces needed to make one whole circle.



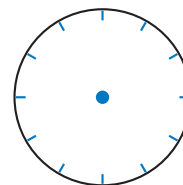
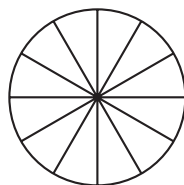
There are 3 one thirds in one.

$$1 \div \frac{1}{3} = 3$$

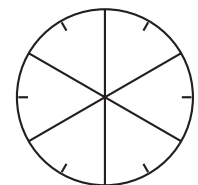
We can use the image of a clock face to help us sketch models for twelfths and sixths. We draw a circle and make twelve tick marks where the numbers 1 through 12 are positioned. To show twelfths, we draw segments from the center of the circle to each of the tick marks. To show sixths, we draw segments only to the tick marks for 2, 4, 6, 8, 10, and 12.



twelfths



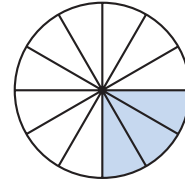
sixths



Example 4

Represent Make a sketch to show the division $\frac{1}{4} \div \frac{1}{12}$. What is the quotient?

We draw a circle divided into twelfths. Then we lightly shade $\frac{1}{4}$ of the circle. To find how many $\frac{1}{12}$ s are in $\frac{1}{4}$, we count the number of $\frac{1}{12}$ s in the shaded portion of the circle. We find that the quotient is **3**.



$$\frac{1}{4} \div \frac{1}{12} = 3$$

Lesson Practice

Represent Make sketches to solve problems **a** and **b**.

- How many one sixths are in one half?
- How many one twelfths are in one third?

Find each quotient. Try answering the problems mentally.

c. $\frac{2}{3} \div \frac{2}{3}$

d. $1 \div \frac{1}{4}$

e. $\frac{2}{3} \div \frac{1}{3}$

f. $1 \div \frac{1}{2}$

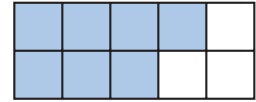
Written Practice

Distributed and Integrated

- Analyze** Mariah's rectangular garden is twice as long as it is wide.
(53) Her garden is 10 feet wide.
 - What is the perimeter of her garden?
 - What is the area of the garden?
- * Multiple Choice** In which of these numbers does the 1 mean $\frac{1}{10}$?
(68) **A** 12.34 **B** 21.43 **C** 34.12 **D** 43.21
- Arrange these numbers in order from least to greatest:
(69, 71) $1, 0, \frac{1}{2}, 0.3$
- * Two quarts of juice is how many ounces of juice?**
(85)

5. a. A quarter is what fraction of a dollar?
(21, 30)
- b. How many quarters equal 1 dollar?
- c. How many quarters equal 3 dollars?

6. Name the shaded portion of this rectangle as a fraction, as a decimal number, and as a percent.
(71)



7. **Multiple Choice** If $a = 3$, then $2a + 5$ equals which of the following?
(17)

A 10 **B** 11 **C** 16 **D** 28

8. \overline{AC} is 84 millimeters. \overline{AB} is one fourth of \overline{AC} . Find \overline{BC} .
(61)



- *9. Write a fraction equal to $\frac{1}{2}$ that has a denominator of 6. Then write a fraction equal to $\frac{1}{3}$ that has a denominator of 6. Subtract the second fraction from the first fraction.
(79)
- *10. a. Find the common factors of 20 and 50.
(82)
- b. Use the GCF of 20 and 50 to reduce $\frac{20}{50}$.

*11. $\frac{3}{5}$ of 4
(86)

*12. $\frac{1}{2} \div \frac{1}{12}$
(87)

13. $3\frac{7}{8} - 1\frac{1}{8}$
(81)

14. $2250 \div 50$
(54)

15. $5 \overline{)225}$
(26)

16. $\begin{array}{r} 5365 \\ 428 \end{array}$
(6)

17. $4 \overline{) \$8.20}$
(34)

18. $20^2 - \sqrt{100}$
(78)

3997

19. $\begin{array}{r} \$12.75 \\ \times \quad 80 \\ \hline \end{array}$
(29)

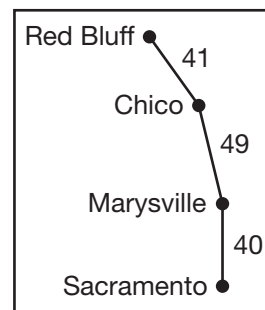
659
7073
+ 342

- *20. Divide 100 by 8 and write the quotient as a mixed number. Then rewrite the quotient by reducing the fraction part of the mixed number.
(58, 81)
- *21. How many one eighths are in one fourth?
(87)
- *22. Since $\frac{2}{3} + \frac{2}{3} + \frac{2}{3} = 2$, how many $\frac{2}{3}$ s are in 2?
(87)

23. The map below shows the number of miles between towns. Use this map to answer problems **a–b**.

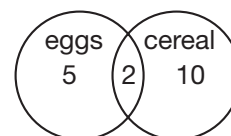
(35,
Inv. 5)

- a. The distance from Marysville to Red Bluff is how many miles greater than the distance from Marysville to Sacramento?
- b. Allen was traveling from Sacramento to Chico. When he was halfway to Marysville, how far did he still have to go to get to Chico?



***24.** Kenji asked 20 classmates whether they ate eggs or cereal for breakfast. He displayed the answers in the Venn diagram at right. Use this information to answer parts **a–e**.

(Inv. 7)



- a. How many students ate cereal for breakfast?
 - b. How many students ate eggs for breakfast?
 - c. How many students ate both cereal and eggs for breakfast?
 - d. Altogether, how many students ate eggs or cereal or both?
 - e. How many students ate neither eggs nor cereal for breakfast?
- 25.** One April day in Norfolk, Virginia, the highest temperature was 8° greater than the lowest temperature. The highest temperature that day was 57°F . What was the lowest temperature?
- (27)
- 26.** Danchelle's family planned an 8-hour car trip. They planned to end their 8-hour trip at midnight. Instead they ended their trip after they had driven only 5 hours 32 minutes. What time did they stop?
- (35)
- *27.** On Sunday evening, Levon studied for $\frac{3}{4}$ of an hour and read a mystery book for $1\frac{3}{4}$ hours. How many hours did Levon spend studying and reading on Sunday evening?
- (75)
- *28.** Randy made $3\frac{1}{4}$ dozen oatmeal cookies, and his sisters and their friends ate $\frac{3}{4}$ of a dozen cookies after they arrived home from school. How many dozen cookies were not eaten?
- (75)

29. **Estimate** The highest-scoring regular season game in the National Basketball Association happened in 1983 when the Denver Nuggets defeated the Detroit Pistons 186–184. About how many points were scored during that game altogether?

*30. The temperature at the top of Mt. Wilson was recorded at 3-hour intervals as shown in the table below. Display the data in a line graph.

Temperature on
Mt. Wilson

Time	Temperature
6:00 a.m.	4°C
9:00 a.m.	6°C
Noon	9°C
3:00 p.m.	11°C
6:00 p.m.	8°C

Early Finishers
Real-World Connection

Stacey has $\frac{2}{3}$ of a liter of juice to serve her friends. If each serving is $\frac{1}{6}$ of a liter, how many servings can she make with the juice she has? Make a sketch to show how you solved the problem.

• Transformations

Power Up

facts

Power Up H

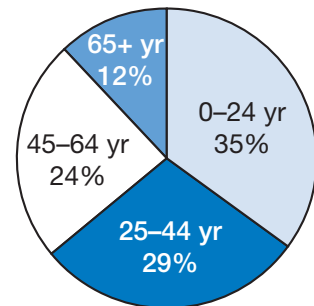
mental math

- Number Sense:** $5(30 + 4)$
- Number Sense:** $5(34)$
- Number Sense:** Reduce the fractions $\frac{3}{12}$, $\frac{10}{12}$, and $\frac{9}{12}$.
- Measurement:** How many ounces are in a pound? How many ounces are in a pint?
- Powers/Roots:** 10^2
- Estimation:** A pencil costs 69¢, a protractor costs \$1.29, and a compass costs \$2.99. Round the cost of each item to the nearest ten cents and then add.
- Percent:** Lacey is studying a list of 800 words for the upcoming spelling bee. She has already studied 25% of the words. How many words has Lacey studied?
- Calculation:** $5^2, - 1, \div 3, + 2, \times 10$

problem solving

Choose an appropriate problem-solving strategy to solve this problem. The circle graph at right is based on year 2004 estimates published by the Census Bureau. The graph shows the percentage of United States residents that belong to each of four age groups. Every 1 percent of the population is equal to nearly 3 million residents. Use that estimate to determine how many more residents are in the 25–44 age group than are in the 45–64 age group.

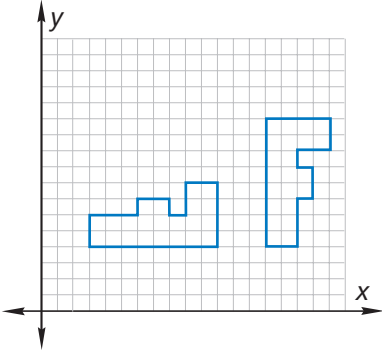
U.S. Population by Age



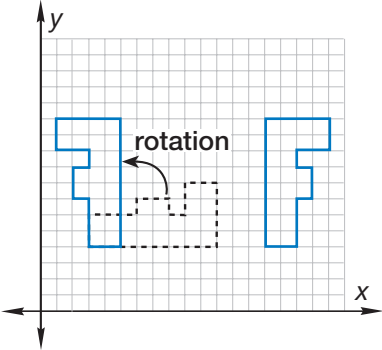
New Concept

Math Language
In mathematics we use the word *transformation* to mean changing the position of a figure through rotation, translation, and reflection.

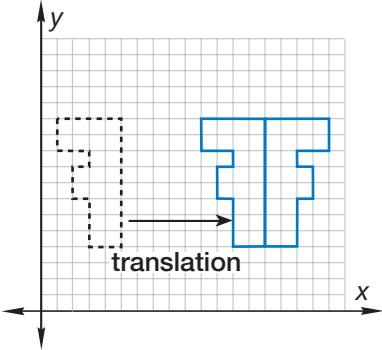
Recall that two figures are *congruent* if one figure has the same shape and size as the other figure. One way to decide whether two figures are congruent is to position one figure “on top” of the other. The two figures below are congruent.



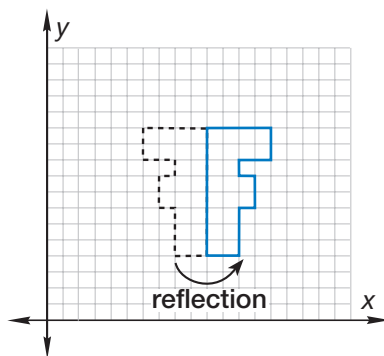
To position the left-hand figure on the right-hand figure, we combine three different kinds of moves. First, we *rotate* (turn) the left-hand figure a quarter turn.



Second, we *translate* (slide) the left-hand figure so that the two figures are back to back.



Third, we *reflect* (flip) the left-hand figure so that it is positioned on top of the right-hand figure.



Recall that the three different kinds of moves we made are called *transformations*. We list them in the following table:

Transformations

Name	Movement
Translation	sliding a figure in one direction without turning the figure
Reflection	reflecting a figure as in a mirror or “flipping” a figure over a certain line
Rotation	turning a figure about a certain point

Rotations can be described by their amount of turn and direction.

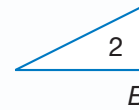


We often use *degrees* to describe the amount of a turn. One full turn is 360° , so a half turn is 180° . If you make a 180° turn, you will face the opposite direction. A quarter turn is 90° . Clockwise is the direction the hands of a clock turn (to the right). Counterclockwise is the opposite direction (to the left).

Activity

Describing Transformations

Describe the transformations that move one figure to the same position of a congruent figure. For example, to move triangle 1 onto triangle 2, we can perform the transformations on the next page. Trace and cut triangles 1 and 2 to help you model the transformations.



1. Translate triangle 1 so that point A is on point B .
2. Then reflect triangle 1 across its vertical side.
3. Then turn triangle 1 a quarter turn counterclockwise.

Example 1

The figure at right is a number 3. Sketch the figure after a 90° clockwise rotation.

A 90° clockwise rotation is a quarter turn to the right.



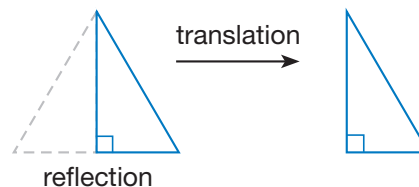
Represent Sketch a reflection of the number 2.

Example 2

Triangles ABC and XYZ are congruent. Name two transformations that would move triangle ABC to the position of triangle XYZ .

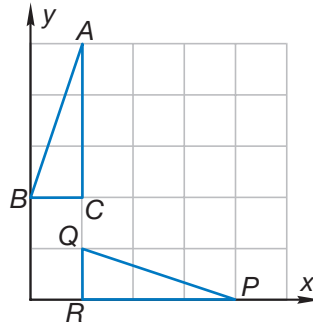


If we **reflect** triangle ABC in line AC , then triangle ABC will have the same orientation as triangle XYZ . Then we **translate** triangle ABC to the position of triangle XYZ .



Example 3

Triangles ABC and PQR are congruent. Describe the two transformations that would move triangle ABC to the position of triangle PQR .



We **rotate** triangle ABC 90° clockwise around point C . Then we **translate** the triangle down 2 units.

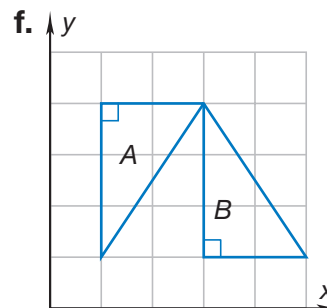
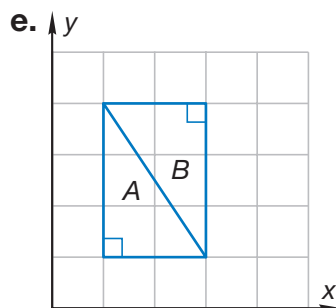
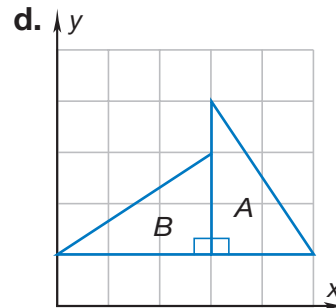
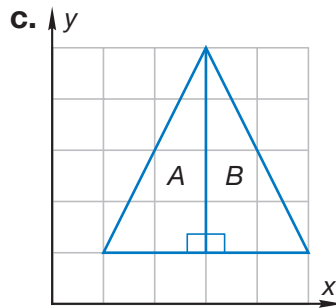
Lesson Practice

a. **Represent** Sketch an uppercase letter R after a reflection in its vertical segment.

R

b. **Represent** Sketch an uppercase letter R after a 90° counterclockwise rotation.

Name the transformation or combination of transformations that could be used to position triangle A on triangle B .



1. Lilah lives $\frac{1}{4}$ mile from school. How far does she travel to and from school each day?
(81)

2. Use the March 2070 calendar to find the date of the first Friday in April 2070.
(28)

MARCH 2070						
S	M	T	W	T	F	S
						1
2	3	4	5	6	7	8
9	10	11	12	13	14	15
16	17	18	19	20	21	22
23	24	25	26	27	28	29
30	31					

3. When the decimal number three and twelve hundredths is subtracted from four and twenty-five hundredths, what is the difference?
(73)

4. a. How many dimes equal \$1?
(21, 30)

b. How many dimes equal \$5?

*5. What number is $\frac{2}{3}$ of 150?
(86)

*6. A half gallon of milk is how many quarts of milk?
(85)

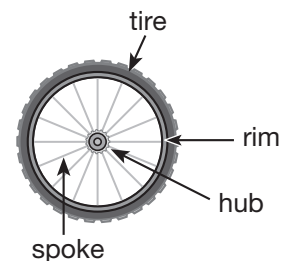
7. **Multiple Choice** Which part of a bicycle wheel is most like a radius?
(53)

A rim

B spoke

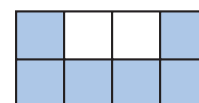
C hub

D tire



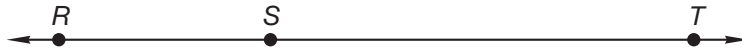
*8. **Analyze** Write a fraction equal to one third that has a denominator of six. Then subtract that fraction from five sixths. Remember to reduce the answer.
(79)

*9. a. What fraction of this rectangle is shaded? Reduce the answer.
(71, 81)



b. What percent of this rectangle is shaded?

10. \overline{RT} is 84 millimeters. \overline{RS} is one third of \overline{RT} . Find \overline{ST} .

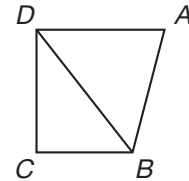


- * 11. Compare: $\frac{3}{5} + \frac{3}{5} + \frac{3}{5} \bigcirc 3 \times \frac{3}{5}$

12. **Multiple Choice** In this drawing, which angle appears to be obtuse?

- A $\angle ABC$
C $\angle BDC$

- B $\angle ABD$
D $\angle DAB$



* 13. $\frac{1}{8} \times 3$

* 14. $\frac{3}{8} \div \frac{1}{8}$

- * 15. a. How many fourths are in 1?

b. $\frac{1}{6} \times 4$

16. $\frac{1}{4} + \frac{1}{4}$

17. $\frac{7}{8} - \frac{1}{8}$

18. $5 - 1\frac{3}{10}$

19. $\$6.57 + 38\text{¢} + \16

20. $421.05 - 125.7$

21. 30^2

- * 22. **Interpret** Use the following school schedule to answer parts a and b:

School Schedule

Subject	Time
Reading	8:00–8:50
Math	8:50–9:40
Recess	9:40–10:10
Language	10:10–10:50
Science	10:50–11:30
Lunch	11:30–12:30

- a. How many total minutes are spent each morning studying reading and language?
- b. If students have 2 hours 10 minutes of class left after lunch, then at what time does school end?

23. 340×607
(56)

24. $9 \overline{) \$7.65}$
(26)

25. Room 16 is 30 feet long and 30 feet wide. What is the floor area of the room?
(72)

*26. **Conclude** Write the next three terms of this arithmetic sequence:
(Inv. 4)

$$\frac{1}{2}, 1, 1\frac{1}{2}, 2, 2\frac{1}{2}, 3, \text{---}, \text{---}, \text{---}, \dots$$

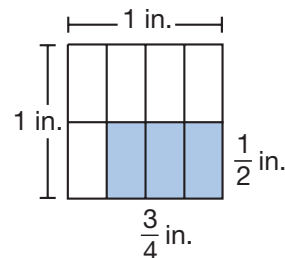
*27. **Analyze** Without looking, Sam chose one marble from a bag containing 2 red marbles, 3 white marbles, and 10 black marbles. Find the probability that the marble Sam chose was black. Write the answer as a reduced fraction.
(57, 81)

*28. a. **Multiple Choice** Which of these Venn diagrams illustrates the relationship between parallelograms (P) and trapezoids (T)?
(45, Inv. 7)



b. **Verify** Explain your answer to the question above.

*29. A portion of this square inch is shaded. What is the area of the shaded rectangle?
(72, 76)



30. The year 1983 was the 200th anniversary of the world's first hot air balloon flight. Fifty-six years after that flight, the world's first pedaled bicycle was invented. In what year was the pedaled bicycle invented?
(49)

• Analyzing Prisms

Power Up

facts

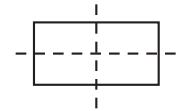
Power Up F

mental math

- Number Sense:** $4(30 + 4)$
- Number Sense:** $4(34)$
- Number Sense:** Reduce the fractions $\frac{4}{12}$, $\frac{6}{12}$, and $\frac{8}{12}$.
- Measurement:** How many ounces are in a pint?
- Measurement:** How many pints are in a quart?
- Measurement:** How many ounces are in a quart?
- Estimation:** Choose the more reasonable estimate for the weight of a pair of scissors: 10 oz or 10 lb.
- Calculation:** $\frac{1}{10}$ of 1000, $-1, \div 9, + 1, \times 4, + 1, \div 7$

problem solving

Choose an appropriate problem-solving strategy to solve this problem. A **line of symmetry** divides a figure into mirror



images. If a rectangle is longer than it is wide, then it has exactly two lines of symmetry: one lengthwise and one widthwise. The lines of symmetry for this rectangle are shown with dashes. On your paper, draw a rectangle that is wider than it is long, and show its lines of symmetry.

New Concept

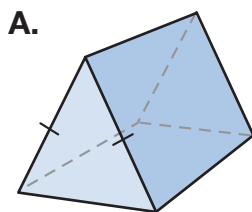
Math Language

A *pyramid* is a three-dimensional solid with one base that can be any polygon. The base of a pyramid is not a face.

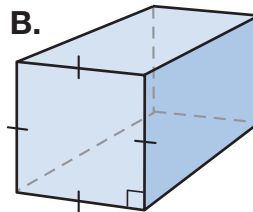
A **prism** is a three-dimensional solid with two congruent bases. These congruent bases are parallel. The shape of each pair of bases can be any polygon. The shape of the base determines the name of the prism. The word *base* does not mean the bottom of the figure. In each figure below, the bases are the front and back of the figure. However, the figures can be turned so that the bases are in different positions.



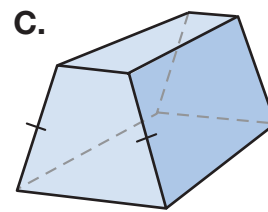
Visit www.SaxonMath.com/Int5Activities for a calculator activity.



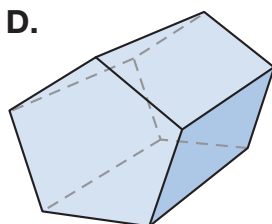
A.
Triangular prism
6 vertices
9 edges



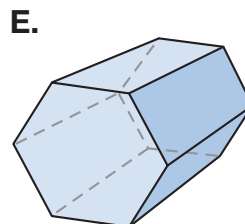
B.
Rectangular prism
8 vertices
12 edges



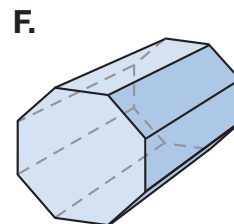
C.
Trapezoidal prism
8 vertices
12 edges



D.
Pentagonal prism
10 vertices
15 edges



E.
Hexagonal prism
12 vertices
18 edges



F.
Octagonal prism
16 vertices
24 edges

Analyze What shape is each pair of bases in prisms *A–F*?
What shape are the faces that are not bases?

Example 1

Which prisms *A–F* have all parallel rectangular faces?

In figure **B**, if we consider the front and back of the rectangular prism as bases, then we see two other pairs of parallel rectangular faces; that is, the top and the bottom faces are parallel, and the left and right faces are also parallel.

Conclude Do all regular polygons have parallel sides? Why or why not?

Example 2

Which prisms *A–F* have congruent rectangular faces?

Prism A has 2 congruent rectangular faces since two sides of the triangular base are the same length.

Prism B has 4 congruent rectangular faces since its bases are squares.

Prism C has 2 congruent rectangular faces since its bases are trapezoids with two same-length sides.

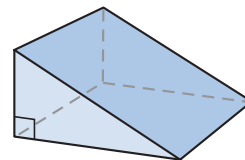
Figures *D*, *E*, and *F* have all rectangular faces congruent since the bases are regular polygons.

Verify When all the faces of a prism are congruent, what is the prism called?

Example 3

Look at this figure. Does it have any perpendicular rectangular faces?

Notice that two sides of the triangular bases are perpendicular. Therefore, two rectangular faces are also perpendicular.

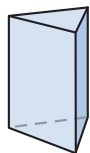


Verify Which prisms *A–F* have perpendicular rectangular faces? Explain why.

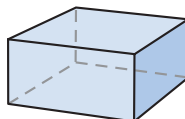
Lesson Practice

Name each type of prism in **a–c**.

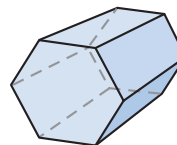
a.



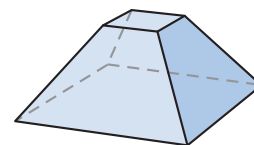
b.



c.



- The prism in problem **a** has how many perpendicular rectangular faces?
- A rectangular prism has how many pairs of parallel faces?
- The prism in problem **c** has how many edges?
- Is this figure a prism? Explain your answer.



Written Practice

Distributed and Integrated

- ⁽³⁵⁾ Thomas Jefferson wrote the Declaration of Independence in 1776. He became president 25 years later. In what year did he become president?

2. **Analyze** Shannon won \$10,000 in the poetry contest. She will be paid \$20 a day until she receives the entire \$10,000.

a. How many days will she be paid \$20?

b. Is that greater or less than a year?

*3. **Multiple Choice** Which of these numbers is divisible by both 4 and 5?

A 15

B 16

C 20

D 25

4. Arrange these numbers in order from least to greatest:

0.5, $\frac{3}{2}$, 1, 0, 1.1

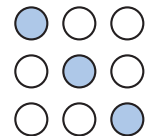
*5. a. How many half-gallon cartons of milk equal 1 gallon?

b. How many half-gallon cartons of milk equal 3 gallons?

6. **Represent** Use digits to write the number one million, three hundred fifty-four thousand, seven hundred sixty.

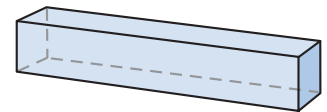
*7. **Analyze** Write a fraction equal to $\frac{1}{2}$ that has a denominator of 6. Then subtract that fraction from $\frac{5}{6}$. Remember to reduce the answer.

8. a. What fraction of the circles is shaded? Reduce the fraction.



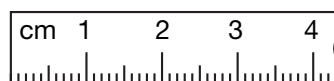
b. What percent of the circles is shaded?

*9. a. Name the shape at right.



b. How many edges does it have?

10. Write the length of the segment below as a number of centimeters and as a number of millimeters.



* 11. $\frac{2}{5}$ of 3
(86)

12. $\frac{2}{5} + \frac{2}{5} + \frac{2}{5}$
(75)

13. $1\frac{1}{4} + 1\frac{1}{4}$
(81)

14. $3\frac{5}{6} - 1\frac{1}{6}$
(81)

15. $42.6 + 49.76 + 28.7 + 53.18$
(73)

16. $\$10 - (57¢ + \$2.48)$
(24, 70)

17. $42 \times 5 \times 36$
(18, 56)

18. $\$6.15 \times 10$
(29)

19. $40 \overline{)2760}$
(54)

20. $4w = 276$
(26)

* 21. $\frac{1}{2} \div \frac{1}{10}$
(87)

* 22. $\frac{1}{2} \times \frac{6}{8}$
(81)

23. Divide 371 by 10 and write the answer with a remainder.
(54)

* 24. Use this information to answer parts **a** and **b**:
(49, Inv. 5)

When Jenny was born, her dad was 29 years old. Her brothers are Noah and Monty. Noah is 2 years older than Jenny and 2 years younger than Monty. Monty is 10 years old.

a. How old is Jenny?

b. How old is Jenny's dad?

25. $\sqrt{25} - \sqrt{9}$
(18)

26. $3^2 + 4^2$
(78)

27. **List** A dime is tossed and then a quarter is tossed. One possible outcome is dime: heads, quarter: tails. List the three other possible outcomes.
(Inv. 6)

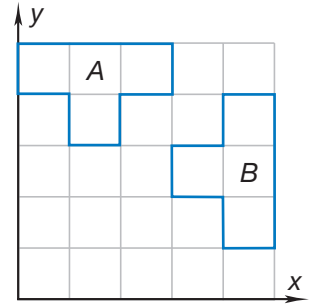
* 28. a. A pint is what fraction of a quart?
(76, 85)

b. What fraction of a gallon is a quart?

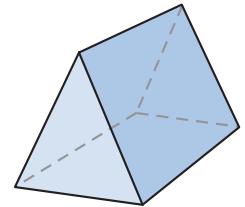
c. What fraction of a gallon is a pint?

d. The answers to parts **a–c** show that one half of one fourth equals what fraction?

- *29. Name two transformations that would move figure *A* to the position of figure *B*.
(88)



- *30. How many triangular and rectangular faces does a triangular prism have?
(89)

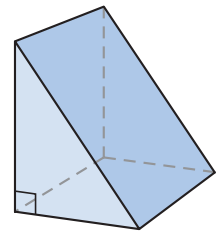


Early Finishers

Real-World Connection

Look at the figure at right.

- What is the name of this figure?
- How many perpendicular rectangular faces does the figure have?
- How many pairs of parallel faces does it have?
- How many edges does it have?



• Reducing Fractions, Part 2

Power Up

facts

mental math

problem solving

Power Up F

- a. **Number Sense:** $6(40 + 6)$
- b. **Number Sense:** $6(46)$
- c. **Number Sense:** Reduce the fractions $\frac{4}{8}$, $\frac{4}{12}$, and $\frac{4}{16}$.
- d. **Fractional Parts:** $\frac{1}{3}$ of 19
- e. **Measurement:** The 1-liter bottle was full of water. Then Quincy poured out 350 mL. How many mL of water were left in the bottle?
- f. **Measurement:** Which U.S. Customary unit is nearly equal to a liter?
- g. **Percent:** Eric and Trey agree to each pay 50% of the \$31 cost of the video game. How much will each boy pay?
- h. **Calculation:** $\frac{1}{4}$ of 36, $+ 1$, $\times 2$, $+ 1$, $\div 3$, $\times 7$, $+ 1$, $\div 2$

At the store, Chris wants to be able to purchase any item that costs from 1¢ to 99¢ using exact change. What is the combination of fewest quarters, dimes, nickels, and pennies that Chris needs?

Focus Strategy: Make it Simpler

Understand We are told that Chris wants to be able to purchase any item that costs from 1¢ to 99¢ using exact change. We are asked to find the combination of fewest quarters, dimes, nickels, and pennies that Chris needs.

Plan We do not want to consider all 99 possible prices from 1¢ to 99¢ individually, so we look for a way to *make it simpler*. We approach the problem by thinking about each coin separately.

Solve We can start with pennies. We know Chris would need 4 pennies to pay for a 4¢ item with exact change. We cannot think of a price for which Chris would need 5 or more pennies, because 5 pennies can always be replaced with only 1 nickel.

Now we think, “What is the greatest number of nickels Chris would need?” We might see that two nickels have a value of 10¢. This means that Chris needs only 1 nickel, because 2 nickels can always be replaced with 1 dime.

For dimes, we think, “3 dimes can always be replaced with 1 quarter and 1 nickel, so Chris needs only 2 dimes.” For quarters, Chris can use 3 quarters for a 75¢ item and 3 quarters plus some additional coins for an item costing from 76¢ to 99¢. So we find that the smallest set of coins Chris needs is **3 quarters, 2 dimes, 1 nickel, and 4 pennies.**

Check We know that our answer is reasonable because the coins we found can make all combinations from 1¢ to 99¢ using fewer pennies than equal a nickel, fewer nickels than equal a dime, and fewer dimes than equal a quarter.

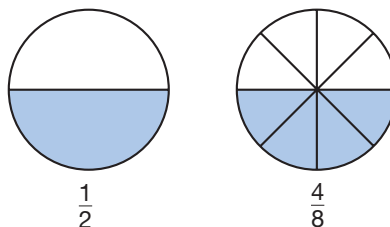
We should also ask ourselves whether there are other possible answers. Can you find another combination of 10 coins that can be used to pay exact change for any price from 1¢ to 99¢?

New Concept

Math Language

A fraction is reduced to *lowest terms* when 1 is the greatest factor that both the numerator and the denominator have in common.

The equivalent fractions pictured below name the same amount. We see that $\frac{4}{8}$ is equivalent to $\frac{1}{2}$.



We can reduce $\frac{4}{8}$ by dividing 4 and 8 by 2.

$$\frac{4 \div 2}{8 \div 2} = \frac{2}{4}$$

If we reduce $\frac{4}{8}$ by dividing both terms by 2, we find that $\frac{4}{8}$ is equal to $\frac{2}{4}$. However, fractions should be reduced to *lowest terms*. The fraction $\frac{2}{4}$ can also be reduced, so we reduce again.

$$\frac{2 \div 2}{4 \div 2} = \frac{1}{2}$$

The fraction $\frac{4}{8}$ reduces to $\frac{2}{4}$, which reduces to $\frac{1}{2}$. We reduce twice to find that $\frac{4}{8}$ equals $\frac{1}{2}$.

We can avoid the need to reduce more than once if we divide by the greatest common factor (GCF) of the terms. The GCF of 4 and 8 is 4. If we reduce $\frac{4}{8}$ by dividing both terms by 4, we reduce only once.

$$\frac{4 \div 4}{8 \div 4} = \frac{1}{2}$$

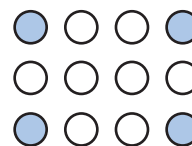
Example 1

Thinking Skill

Connect

How can we change $\frac{2}{3}$ to the equivalent fraction $\frac{8}{12}$?

There are 4 blue marbles and 8 white marbles in a bag. If one marble is taken from the bag without looking, what is the probability that the marble selected is white?



Since 8 of the 12 marbles are white, the probability of selecting a white marble is $\frac{8}{12}$. Since 8 and 12 are divisible by 2, we may reduce $\frac{8}{12}$ by dividing both terms by 2. This gives us $\frac{4}{6}$, which also can be reduced.

Reduce Twice

$$\frac{8 \div 2}{12 \div 2} = \frac{4}{6}$$
$$\frac{4 \div 2}{6 \div 2} = \frac{2}{3}$$

We save a step if we reduce by the GCF of the terms. The GCF of 8 and 12 is 4. If we divide 8 and 12 by 4, then we reduce only once.

Reduce Once

$$\frac{8 \div 4}{12 \div 4} = \frac{2}{3}$$

The probability that the marble selected is white is $\frac{2}{3}$.

Example 2

The value of a dime is 40% the value of a quarter. Write 40% as a reduced fraction.

We first write 40% as the fraction $\frac{40}{100}$. Since the numerator and denominator both end in zero, we know they are divisible by 10.

$$\frac{40 \div 10}{10 \div 10} = \frac{4}{10}$$

Since the terms of $\frac{4}{10}$ are both even, we can continue to reduce by dividing both terms by 2.

$$\frac{4 \div 2}{10 \div 2} = \frac{2}{5}$$

The GCF of 40 and 100 is 20. So we could have reduced $\frac{40}{100}$ in one step by dividing both terms by 20.

$$\frac{40 \div 20}{100 \div 20} = \frac{2}{5}$$

Lesson Practice

Reduce each fraction to lowest terms:

a. $\frac{4}{12}$

b. $\frac{6}{18}$

c. $\frac{16}{24}$

d. $\frac{4}{16}$

e. $\frac{12}{16}$

f. $\frac{60}{100}$

Solve. Reduce each answer to lowest terms.

g. $\frac{7}{16} + \frac{1}{16}$

h. $\frac{3}{4} \times \frac{4}{5}$

i. $\frac{19}{24} - \frac{1}{24}$

Write each percent as a reduced fraction:

j. 25%

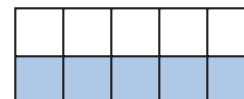
k. 60%

l. 90%

Written Practice

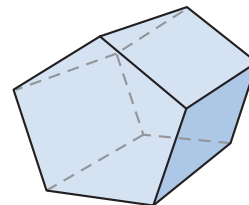
Distributed and Integrated

- This little poem is about what number?
(6)
*I am a number, not 1, 2, or 3.
Whenever I'm added, no difference you'll see.*
- Analyze** Write a fraction equal to $\frac{1}{2}$ and a fraction equal to $\frac{3}{5}$ with denominators of 10. Then add the fractions. Remember to convert the answer to a mixed number.
(75, 79)
- *3.** a. How many quarts of milk equal a gallon?
(85)
b. How many quarts of milk equal 6 gallons?
- Find the sum when the decimal number fourteen and seven tenths is added to the decimal number four and four tenths.
(68, 73)
- Name the shaded portion of this rectangle as a decimal number, as a reduced fraction, and as a percent:
(71)



6. How many vertices does this prism have?

(83)



Refer to rectangle $ABCD$ to answer problems 7–9.

*7. **Multiple Choice** In this rectangle, which segment is parallel to \overline{AB} ?

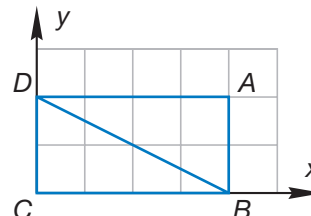
(31, 61)

A \overline{BC}

C \overline{BD}

B \overline{CD}

D \overline{DA}



*8. Classified by angles, what type of triangle is triangle BCD ?

(36, 61)

*9. **Conclude** What transformation would move triangle DAB to the position of triangle BCD ?

(88)

10. $\frac{5}{6} + \frac{5}{6}$

(75)

*11. $\frac{5}{6} \times 2$

(86)

*12. $\frac{2}{5} \div \frac{1}{10}$

(87)

*13. $\frac{1}{12} + \frac{7}{12}$

(41, 90)

14. $6\frac{2}{3} - \left(4 - \frac{1}{3}\right)$

(41, 63)

*15. $\frac{2}{3} \times \frac{3}{4}$

(76, 90)

16. $26.4 + 2.64$

(73)

17. $8.36 - 4.7$

(73)

18. 40^2

(78)

19. $\sqrt{36} + \sqrt{64}$

(78)

20. $480 \div 10$

(54)

21. $5n = 240$

(26)

22. $1 \div \frac{1}{3}$

(87)

23. $\frac{3}{4} \times 3$

(86)

*24. $\frac{3}{5} \times \square = \frac{60}{100}$

(79)

25. The table below lists ways Trevin can earn extra credit in social studies.
(11, Inv. 5) Use the table below to answer parts **a** and **b**.

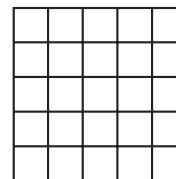
Extra Credit

Magazine report	35 points
TV report	50 points
Book report	75 points
Museum report	100 points

- a.** Trevin has done a book report, two magazine reports, and a TV report. How many points has he earned?
- b.** Trevin wants to earn a total of 400 points. How many more points does he need?
26. **Analyze** A bag contains 3 red marbles, 5 white marbles, 2 blue marbles, and 6 orange marbles. A marble is drawn without looking. Find the probability that the marble is orange. Write the answer as a reduced fraction.
(57, 81)

27. The area of this square is 25 square inches.
(53, 72)

- a.** How long is each side?
- b.** What is its perimeter?



- * 28. **Generalize** What is the next term of the sequence below?
(32, Inv. 4) Describe the pattern in words.



- * 29. **a. List** List the factors of 16 in order from least to greatest.
(78, 84)
- b.** Is the number of factors odd or even?
- c.** What is the median of the factors?
- d.** What is $\sqrt{16}$?

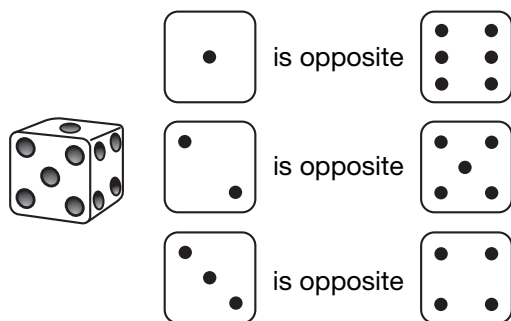
30. **Explain** Yvonne is car shopping. One model she likes costs \$23,460. Another model she likes costs \$24,575. What is a reasonable estimate of the cost difference of those two models? Explain how you found your answer.
(31)

Focus on

• Performing Probability Experiments

In Lesson 57 we used the word *probability* to describe how likely it is that a given event occurs in an experiment. Probabilities are fractions. If we repeat an experiment over and over, we can use probability to predict the number of times an event will occur.

A typical dot cube has six faces marked with dots representing the numbers 1, 2, 3, 4, 5, and 6.



As an experiment, we can roll a dot cube and record the upturned face as an outcome. Because the 6 possible outcomes are equally likely, each outcome must have the same probability. The probabilities of all the outcomes must add up to one, so each outcome has a probability of $\frac{1}{6}$.

$$\frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} = \frac{6}{6} = 1$$

We can add probabilities in order to determine the likelihood of one of a certain collection of outcomes. For example, the probability that the upturned face will be an even number is the sum of the probabilities of rolling a 2, a 4, or a 6.



$$\frac{1}{6} + \frac{1}{6} + \frac{1}{6} = \frac{3}{6} = \frac{1}{2}$$

1. What is the probability that the upturned face is either 1 or 6?
2. What is the probability that the upturned face is less than 5?

If we roll our dot cube repeatedly, we can predict how many times certain events will occur. Our guess is based on the part-of-a-group meaning of a fraction. Suppose we rolled our dot cube 24 times. Since all the outcomes have a probability of $\frac{1}{6}$, we predict that we would roll the number 2 (or any other particular number) one sixth of 24 times. This means we divide 24 by 6.

$$24 \div 6 = 4 \text{ times}$$

Because three faces show even numbers, we would predict that we would roll an even number 3×4 times or 12 times. These are only guesses; the actual number of times that an event will occur cannot be predicted with certainty.

-  **Predict** If a standard dot cube is rolled 60 times, how many times would you predict that the upturned face will be 1? Explain your answer.
-  **Predict** If a standard dots cube is rolled 60 times, how many times would you predict that the upturned face will be either 1 or 6? Explain your answer.




Activity 1

Probability Experiment 1

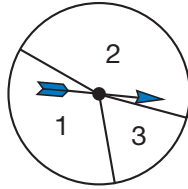
Materials needed:

- **Lesson Activity 30**
- dot cube

Roll the dot cube 24 times, and tally each outcome in the frequency table on **Lesson Activity 30**. Use the tallies you recorded to answer problems 5–8.

5. Use the tallies to complete the “Frequency” column on your table.
6. Which of the six outcomes occurred more frequently than you would guess?
7. How many times was the upturned face even?
8.  **Predict** Based on your table, predict what the next roll will be.

We can perform probability experiments repeatedly to estimate probabilities that we do not know how to calculate. Suppose Serena constructs a spinner with 3 regions by dividing up a circle without any definite plan. The spinner she makes is shown below.



To estimate the fraction of the whole that each region takes up, she spins the spinner 50 times. She presents the results in a **relative frequency table**. In the last column Serena records the number of times each outcome occurred as the numerator of a fraction with denominator 50.

Outcome	Tally	Relative Frequency
1		$\frac{17}{50}$
2		$\frac{28}{50}$
3		$\frac{5}{50}$

Since 17 of 50 spins stopped on 1, Serena estimates the probability of outcome 1 to be $\frac{17}{50}$. In other words, Serena guesses on the basis of her spins that region 1 takes up about $\frac{17}{50}$ of the entire circle. Similarly, she estimates the probability of outcome 2 as $\frac{28}{50}$ and the probability of outcome 3 as $\frac{5}{50}$.

- Analyze** Because $\frac{28}{50} > \frac{17}{50}$, outcome 2 seems more likely than outcome 1. Because $\frac{17}{50} > \frac{5}{50}$, outcome 1 seems more likely than outcome 3. If you just looked at the spinner and not at the table, would you make the same statements? Why?
- Evaluate** Do you think $\frac{28}{50}$ overestimates the true probability of stopping on 2 or underestimates it? Give supporting reasons.

 **Activity 2****Probability Experiment 2**

Materials needed:

- **Lesson Activity 30**
- cardboard or posterboard
- scissors
- markers

For this activity, work with a partner.

Make 5 equal-sized squares. While your eyes are closed, have your partner write either “C,” “A,” or “T” on each square. (Each letter must be used at least once.) Then have your partner mix up the squares on a table. With your eyes still closed, choose a square and have your partner tally the outcome on **Lesson Activity 30**. Repeat the process of mixing, choosing, and recording 30 times. Remember to keep your eyes closed.

11. Use the tallies to complete the “Relative Frequency” column on your table. (Remember, the denominator of each relative frequency is the number of times the experiment was performed.)

Letter	Tally	Relative Frequency
C		
A		
T		

12. **Estimate** From your relative frequency table, estimate the probability that the letter you choose is a T.
13. **Predict** If your partner had written a letter just once, about how many times would you expect to choose it out of 30?
14. **Predict** If your partner had written a letter twice, about how many times would you expect to choose it out of 30?
15. **Predict** If your partner had written a letter three times, about how many times would you expect to choose it out of 30?
16. **Analyze** Which of the letters do you think your partner wrote once? Twice? Three times?



- a. If an experiment has N outcomes and they are equally likely, then each has probability $\frac{1}{N}$. Thus, if we flip a coin, which has two equally likely outcomes, the probability of the coin landing heads up is $\frac{1}{2}$ and the probability of the coin landing tails up is $\frac{1}{2}$.

Suppose we write each letter of the alphabet on an identical tile and turn the tiles over. If we select one tile at random, what is the probability that the tile is the letter E? What is the probability that the tile is a vowel? What is the probability that the tile is a consonant?

- b. **Predict** A bag contains 20 colored tiles that are red, yellow, or green. A tile was picked from the bag and replaced 30 times. The table below shows how many times each color was picked.

Outcome	Tally	Relative Frequency
Red		
Yellow		
Green		

Use the outcomes shown in the table to predict how many tiles of each color are in the bag.