• Multiplying Decimal Numbers by 10, by 100, and by 1000

Power Up

**facts**   Power Up K

mental math

a. **Estimation:** Estimate $7\frac{4}{5} \div 3\frac{3}{4}$ by rounding each mixed number to the nearest whole number and then dividing.

b. **Estimation:** Choose the more reasonable estimate for the temperature on a cold winter day: 31°F or 31°C.

c. **Measurement:** How many meters are in one kilometer? … in one tenth of a kilometer?

d. **Percent:** What is 50% of $10? … 25% of $10? … 10% of $10?

e. **Percent:** The calculator is on sale for 25% off the regular price of $10. What is the sale price?

f. **Number Sense:** Write these numbers in order from least to greatest: 0.02, 0.20, 0.19.

g. **Calculation:** $\frac{1}{3}$ of 60, × 2, + 2, ÷ 6, × 4, + 2, ÷ 2

h. **Roman Numerals:** Compare: XLIV $\bigcirc$ 45

problem solving

Choose an appropriate problem-solving strategy to solve this problem. Baseboard is a material that can be placed where the floor meets a wall. The outer edges of the scale drawing to the right indicate walls. The open spaces in the wall represent doors where baseboard is not used. Use your ruler to determine how many meters of baseboard are needed for the room represented by the scale drawing.

Bedroom

1 cm = 2 m
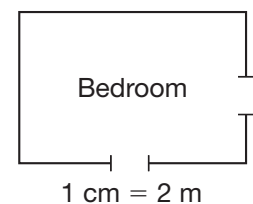
Thinking Skill

Verify

When you multiply a positive number by 10, will the product be greater than that number or less than that number?

Each place in our decimal number system is assigned a particular value. The value of each place is 10 times greater each time we move one place to the left. So when we multiply a number by 10, the digits all shift one place to the left. For example, when we multiply 34 by 10, the 3 shifts from the tens place to the hundreds place, and the 4 shifts from the ones place to the tens place. We fill the ones place with a zero.

3   4 .

3   4   0 .   (10 × 34 = 340)

Shifting digits to the left can help us quickly multiply decimal numbers by 10, 100, or 1000. Here we show a decimal number multiplied by 10.

0 . 3   4

3 . 4   (10 × 0.34 = 3.4)

We see that the digit 3 moved to the other side of the decimal point when it shifted one place to the left. The decimal point holds steady while the digits move. Although it is the digits that change places when the number is multiplied by 10, we can produce the same result by moving the decimal point in the opposite direction.

Shift the digits   **or**   Shift the decimal point
to the left.   to the right.

0 . 3   4   0 . 3   4

3 . 4   (10 × 0.34 = 3.4)   3 . 4

When we multiply by 10, we may simply shift the decimal point one place to the right.

Since 100 is 10 × 10, multiplying by 100 is like multiplying by 10 *twice.* When we multiply by 100, we may shift the decimal point *two* places to the right.

Since 1000 is 10 × 10 × 10, we may shift the decimal point *three* places to the right when we multiply by 1000.

The number of places we shift the decimal point is the same as the number of zeros we see in 10, 100, or 1000.

Example

Multiply: 1.234 × 100

To multiply mentally by 100, we may shift the decimal point two places to the right. The product is **123.4.**

$$1.234 \times 100 = 123.4$$

Generalize Why did we shift the decimal point two places to the right?

Lesson Practice Multiply:

a. 1.234 × 10   **b.** 1.234 × 1000   **c.** 0.1234 × 100

d. 0.345 × 10   **e.** 0.345 × 100   **f.** 0.345 × 1000

g. 5.67 × 10   **h.** 5.67 × 1000   **i.** 5.67 × 100

Written Practice

Distributed and Integrated

1. In three classrooms there were 23 students, 25 students, and 30 students.
(50) If the students in the three classrooms were rearranged so that there were an equal number of students in each room, how many students would there be in each classroom?

2. Composer Duke Ellington was born in 1899. Composer John Williams
(35) was born 33 years later. When was John Williams born?

3. a. Write the reduced fraction equal to 25%.
(71, 90)

b. Write the reduced fraction equal to 50%.

4. a. List the first six multiples of 6.
(15)

b. List the first four multiples of 9.

c. Which two numbers appear in both lists?

5. **Connect** Name the shaded portion of this square as a
(71) percent, as a decimal number, and as a reduced fraction.
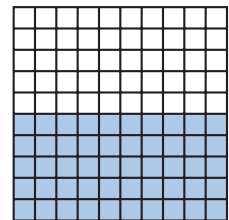
6. Multiple Choice Which is the shape of a basketball?
(83)

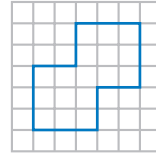   **A** cylinder   **B** sphere   **C** cone   **D** circle

7. How many months are in $1\frac{1}{2}$ years?
(28)

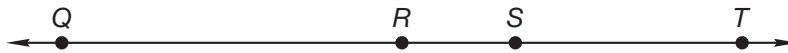**8. a.** How many units long is the perimeter of this shape?
(53, 72)

   b. How many square units is the area of this shape?

9. *QR* is 45 mm. *RS* is one third of *QR*. *QT* is 90 mm. Find *ST*.
(61)

$$\xleftarrow{\qquad \overset{Q}{\bullet} \qquad\qquad\qquad \overset{R}{\bullet} \quad \overset{S}{\bullet} \qquad\quad \overset{T}{\bullet} \qquad}\rightarrow$$

For problems **10** and **11,** multiply mentally by shifting the decimal point.

*10. 1.23×10
(111)

*11. 3.42×1000
(111)

*12. (**Represent**) Use words to name this sum:
(68, 106)

$$15 + 9.67 + 3.292 + 5.5$$

*13. $4.3 - 1.21$
(102)

*14. 0.14×0.6
(110)

*15. 48×0.7 *
(109)

*16. 0.735×10^2
(78, 111)

17. (**Analyze**) Write a fraction equal to $\frac{3}{4}$ that has the same denominator
(75, 79)   as $\frac{3}{8}$. Then add the fraction to $\frac{3}{8}$. Remember to convert your answer to a mixed number.

18. $16\overline{)4000}$
(94)

*19. $\$18.00 \div 10$
(54)

20. $\dfrac{7}{11}$
(91)
$+\ \dfrac{8}{11}$

21. $3\dfrac{7}{12}$
(90)
$+\ \dfrac{1}{12}$

22. $5\dfrac{9}{10}$
(81)
$-\ 5\dfrac{3}{10}$

23. $\dfrac{7}{2} \times \dfrac{1}{2}$
(91)

24. $\dfrac{2}{3} \div \dfrac{1}{4}$
(96)

25. $3 \div \dfrac{3}{4}$
(96)

26. Compare: $\sqrt{9} + \sqrt{16}\ \bigcirc\ \sqrt{9 + 16}$
(78)

*27. The names of two of the 12 months begin with the letter A. What
(107) percent of the names of the months begin with the letter A?

*28. Elizabeth studied this list of flights between Los Angeles and
(108) Philadelphia. Refer to this list to answer parts **a** and **b**.

Los Angeles to Philadelphia	
Depart	**Arrive**
6:15 a.m.	2:34 p.m.
10:10 a.m.	6:33 p.m.
12:56 p.m.	9:15 p.m.
3:10 p.m.	11:19 p.m.

Philadelphia to Los Angeles	
Depart	**Arrive**
7:55 a.m.	10:41 a.m.
10:00 a.m.	12:53 p.m.
1:30 p.m.	4:17 p.m.
5:40 p.m.	8:31 p.m.

a. Elizabeth wants to arrive in Philadelphia before 8 p.m. However,
she does not want to wake up very early to catch a flight. Which
departure time is Elizabeth likely to choose?

b. For her return flight, Elizabeth would like to leave as late as
possible and still arrive in Los Angeles by 9:00 p.m. Which
departure time is Elizabeth likely to choose?

29. A classroom bookshelf contains 27 books. Eleven of the books are
(49) reference books. Five of the books are fiction books. How many books
on the bookshelf are not reference or fiction books?

30. At Franklin Elementary School, the first recess of the morning lasts for $\frac{1}{2}$
(76) of $\frac{1}{2}$ of an hour. What fraction of an hour is the length of the first recess?
How many minutes long is that recess?

Early Finishers
Real-World Connection

To view a slide of an amoeba, Kymma sets a microscope to enlarge
objects to 100 times their actual size.

a. If the actual diameter of the amoeba is 0.095 mm, then what is its
diameter as seen through the microscope?

b. If Kymma sets the microscope to enlarge objects to 10 times their
actual size, what would the diameter of the amoeba appear to be
for that setting?

c. If Kymma sets the microscope to enlarge objects to 1000 times
their actual size, what would the diameter of the amoeba appear to
be in centimeters?

• Finding the Least Common Multiple of Two Numbers

Power Up

facts   Power Up K

mental math

a. Estimation: Estimate the cost of 98 tickets that cost $2.50 each.

b. Measurement: Elsa was feeling ill. Her fever was 100.7°F. How many degrees was Elsa's fever above her normal temperature of 98.6°F?

c. Measurement: The liquid medicine dropper can hold 1 milliliter of liquid. How many full droppers equal half a liter?

d. Fractional Parts: What is $\frac{1}{10}$ of 30? … $\frac{3}{10}$ of 30? … $\frac{9}{10}$ of 30?

e. Probability: The box contains equal amounts of three flavors of dog treats: peanut butter, vegetable, and chicken. If Grey pulls one dog treat from the box without looking, what is the probability it will *not* be chicken?

f. Geometry: If the area of a square is 9 cm^2, what is the length of each side?

g. Calculation: $\sqrt{100}$, ÷ 2, × 7, + 1, ÷ 6, × 4, ÷ 2

h. Roman Numerals: Compare: 96 ◯ XCIV

problem solving

Choose an appropriate problem-solving strategy to solve this problem. Fernando dropped a rubber ball and found that each bounce was half as high as the previous bounce. He dropped the ball from 8 feet, measured the height of each bounce, and recorded the results in a table. Copy this table and complete it through the fifth bounce.

Heights of Bounces

First	4 ft
Second	
Third	
Fourth	
Fifth	

New Concept

Here we list the first few multiples of 4 and 6:

Multiples of 4: 4, 8, ⑫, 16, 20, ㉔, 28, 32, ㊱, …

Multiples of 6: 6, ⑫, 18, ㉔, 30, ㊱, …

We have circled the multiples that 4 and 6 have in common. The smallest number that is a multiple of both 4 and 6 is 12.

The smallest number that is a multiple of two or more numbers is called the **least common multiple** of the numbers. The letters *LCM* are sometimes used to stand for *least common multiple.*

Example

Find the least common multiple (LCM) of 6 and 8.

We begin by listing the first few multiples of 6 and 8. Then we circle the multiples they have in common.

Multiples of 6: 6, 12, 18, ㉔, 30, 36, 42, ㊽, …

Multiples of 8: 8, 16, ㉔, 32, 40, ㊽, …

As we see above, the *least* of the common multiples of 6 and 8 is **24.**

Activity

Prime Numbers on a Hundred Number Chart

Material needed:

- **Lesson Activity 21**

The first prime number is 2 because 2 has two different factors, but 1 has only one factor. Every even number greater than 2 (such as 4, 6, 8, and so on) is a composite number. Since all even numbers are multiples of 2, they have at least 3 factors—the number itself, the number 1, and 2. On a hundred number chart, we can find the prime numbers and cross out the composite numbers, which are all multiples of prime numbers.

	②	3		5		7		9	
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

On this hundred number chart, we circled 2 and began crossing out multiples of 2. On **Lesson Activity 21,** find all the prime numbers. Circle 2 and cross out the multiples of 2. Then circle the next prime number, 3, and cross out the remaining multiples of 3. Then move on to 5, and continue the process until you have found all the prime numbers less than 100.

Lesson Practice

Find the least common multiple (LCM) of each pair of numbers:

a. 2 and 3   **b.** 3 and 5   **c.** 5 and 10

d. 2 and 4   **e.** 3 and 6   **6**
and 10

g. The denominators of $\frac{5}{8}$ and $\frac{3}{10}$ are 8 and 10. What is the least common multiple of 8 and 10?

h. Use color tiles to make factor arrays for 13 and 15. Which number is prime and which number is composite?

Written Practice

Distributed and Integrated

1. **Estimate** A small car weighs about one ton. Most large elephants
(77) weigh four times that much. About how many pounds would a large elephant weigh?

*2. **Estimate** At one time, the Arctic Ocean was almost completely
(74) covered by a polar ice cap, which measured up to 10 feet thick. About how many inches thick was the polar ice cap at that time?

*3. What is the total cost of 10 movie tickets priced at $5.25 each?
(21, 111)

4. Which digit in 375.246 is in the hundredths place?
(68)

5. (**Represent**) Draw a pentagon. Then draw a reflection of your figure.
(32, 88)

6. Write 12.5 as a mixed number.
(71)

7. (**Connect**) Name the shaded portion of the square at right
*(71)* as a percent, as a decimal number, and as a reduced fraction.
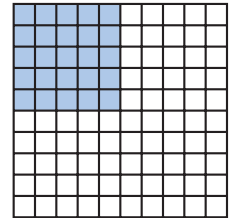
8. Name the shape of an aluminum can.
(83)

9. Stefano practiced playing the trombone for 20 minutes on Monday. On
(49) Wednesday he practiced for 10 minutes more than he did on Monday. On Friday he practiced for 5 fewer minutes than on Wednesday. How many minutes did Stefano practice on Friday?

*10. Find the least common multiple (LCM) of 6 and 9.
(112)

11. If $\overline{OM}$ measures 15 mm, then what is the measure of $\overline{LN}$?
(53, 61)
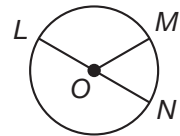
*12. *WX* is 4.2 cm. *XY* is 3 cm. *WZ* is 9.2 cm. Find *YZ.*
(61, 102)

13. $4.38 + 7.525 + 23.7 + 9$
(99)

*14. $5 - (4.3 - 0.21)$ ***15.** 3.6×40 ***16.** 0.15×0.5
(24, 102)   *(109)*   *(110)*

*17. 10×0.125 **18.** $4w = 300$ **19.** $40\overline{)3000}$
(111)   *(26)*   *(54)*

20. $25\overline{)3300}$
(94)

21. $3\frac{3}{7} + \left(5 - 1\frac{2}{7}\right)$
(63, 75)

22. $1\frac{1}{2} - \left(3 \times \frac{1}{2}\right)$
(41, 86)

23. Write fractions equal to $\frac{1}{4}$ and $\frac{2}{3}$ that have denominators of 12. Then
(79) subtract the smaller fraction from the larger fraction.

*24. Use this grid to answer parts **a** and **b**.
(32,
Inv. 8)
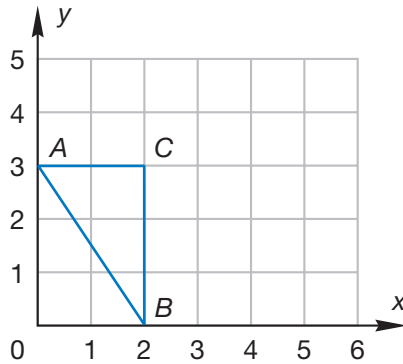
a. Name the coordinates of the vertices of triangle *ABC*.

b. If triangle *ABC* were rotated 90° clockwise around point *C*, then what would be the coordinates of vertex *A*?

25. Compare: $3^2 + 4^2 \bigcirc 5^2$
(78)

*26. Find the percent equivalent of $\frac{1}{8}$ by multiplying 100% by $\frac{1}{8}$. Write the
(107) result as a mixed number with the fraction reduced.

27. The lowest temperature ever recorded in North Dakota was −60°F.
(98) In Montana the lowest temperature ever recorded was −70°F. Is a temperature of −60°F warmer or colder than a temperature of −70°F? How many degrees warmer or colder?

*28. Karen's flight schedule between Oklahoma City and Indianapolis is
(108) shown below. Refer to this schedule to answer parts **a–c.**

Flight	Time	Departure	Arrival
FLIGHT 41 Thu, Aug 22	6:11 a.m. to 8:09 a.m. plane change	Oklahoma City (OKC)	Chicago (ORD)
FLIGHT 11 Thu, Aug 22	9:43 a.m. to 10:38 a.m.	Chicago (ORD)	Indianapolis (IND) Total duration: 4 h 27 min
FLIGHT 327 Thu, Aug 29	9:58 a.m. to 11:03 a.m. plane change	Indianapolis (IND)	St Louis (STL)
FLIGHT 337 Thu, Aug 29	12:04 p.m. to 1:33 p.m.	St Louis (STL)	Oklahoma City (OKC) Total duration: 3 h 35 min

a. Flight 41 of Karen's trip to Indianapolis takes her to Chicago. How
much time is in the schedule for changing planes in Chicago?

b. The times listed in the schedule are gate-to-gate times, from the
time the plane pushes away from the gate at departure to the time
the plane pulls into the gate at arrival. Find the total of the
gate-to-gate times for the two flights from Oklahoma City to
Indianapolis.

c. ✎ **Explain** The total sum of the gate-to-gate times for the two
return flights to Oklahoma City is how many minutes less than for
the outbound flights? What might account for the difference in
travel time?

29. In Ms. Adrian's math class, the students spent $\frac{1}{12}$ of an hour correcting
(41) homework and $\frac{5}{12}$ of an hour working at the board. In simplest form,
what fraction of an hour did the students spend doing those tasks?

30. Lionel chopped $\frac{3}{4}$ of a cup of celery, but he needed to use only $\frac{1}{2}$ of that
(75) amount in a cream soup recipe. What amount of chopped celery did
the recipe require?

• Writing Mixed Numbers as Improper Fractions

Power Up

facts   Power Up K

mental math

a. **Measurement:** The swimming pool holds a maximum of 12,000 gallons of water. Carole has already put about 5500 gallons into the pool. About how many more gallons of water are needed to fill the pool?

b. **Number Sense:** Simplify the fractions $\frac{8}{12}$, $\frac{9}{12}$, and $\frac{15}{12}$.

c. **Percent:** 25% of 12

d. **Percent:** 50% of 19

e. **Percent:** 75% of 12

f. **Geometry:** A hectare is an area of land equivalent to a square that is 100 meters on each side. How many hectares is a field that is 200 meters on each side?

g. **Calculation:** $\frac{1}{6}$ of 24, × 5, + 1, ÷ 3, × 8, − 2, ÷ 9

h. **Roman Numerals:** Compare: MD ◯ 2000

problem solving

Choose an appropriate problem-solving strategy to solve this problem. Blake is saving money for a new telescope. In January Blake saved $10. In the months February through May, he saved $35 each month. By the end of August, Blake will have doubled the total amount of money he had at the end of May. At that time, will Blake have enough money to purchase a telescope that costs $280? Explain your reasoning.

The picture below shows $1\frac{1}{2}$ shaded circles. How many half circles are shaded?
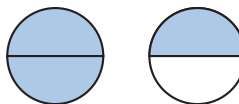
Three halves are shaded. We may name the number of shaded circles as the mixed number $1\frac{1}{2}$ or as the improper fraction $\frac{3}{2}$.

$$1\frac{1}{2} = \frac{3}{2}$$

We have converted improper fractions to mixed numbers by dividing. In this lesson we will practice writing mixed numbers as improper fractions. We will use this skill later when we learn to multiply and divide mixed numbers.

To help us understand changing mixed numbers into fractions, we can draw pictures. Here we show the number $2\frac{1}{4}$ using shaded circles:
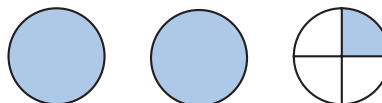
To show $2\frac{1}{4}$ as an improper fraction, we divide the whole circles into the same-size pieces as the divided circle. In this example we divide each whole circle into fourths.
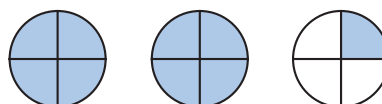
Now we count the total number of fourths that are shaded. We see that $2\frac{1}{4}$ equals the improper fraction $\frac{9}{4}$.

Example 1

Name the number of shaded circles as an improper fraction and as a mixed number.
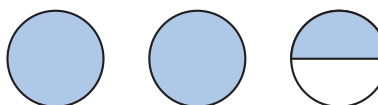
To show the improper fraction, we divide the whole circles into the same-size pieces as the divided circle (in this case, halves). The improper fraction is $\frac{5}{2}$. The mixed number is $2\frac{1}{2}$.
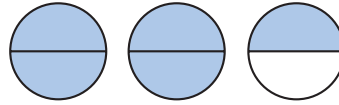
$$\frac{2}{2} + \frac{2}{2} + \frac{1}{2} = \frac{5}{2} = 2\frac{1}{2}$$

Example 2

Change $2\frac{1}{3}$ to an improper fraction.

One way to find an improper fraction equal to $2\frac{1}{3}$ is to draw a picture that illustrates $2\frac{1}{3}$.
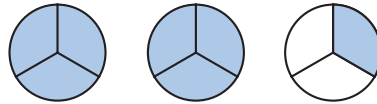
We have shaded 2 whole circles and $\frac{1}{3}$ of a circle. Now we divide each whole circle into thirds and count the total number of thirds.
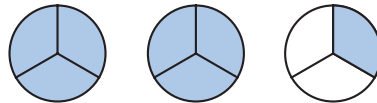
$$\frac{3}{3} + \frac{3}{3} + \frac{1}{3} = \frac{7}{3}$$

We see that seven thirds are shaded, so an improper fraction equal to $2\frac{1}{3}$ is $\frac{7}{3}$**.**

It is not necessary to draw a picture. We could remember that each whole is $\frac{3}{3}$. So the 2 of $2\frac{1}{3}$ is equal to $\frac{3}{3} + \frac{3}{3}$, which is $\frac{6}{3}$. Then we add $\frac{6}{3}$ to $\frac{1}{3}$ and get $\frac{7}{3}$.

Lesson Practice

For problems **a–c,** name the number of shaded circles as an improper fraction and as a mixed number:

a.

b.

c.
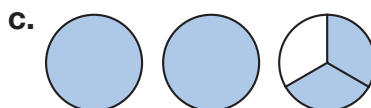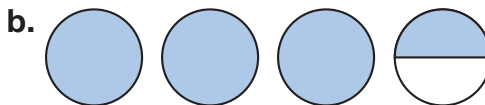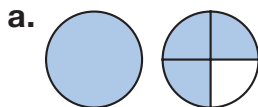
Change each mixed number to an improper fraction:

d. $4\frac{1}{2}$ **e.** $1\frac{2}{3}$ **f.** $2\frac{3}{4}$ **g.** $3\frac{1}{8}$

Written Practice *Distributed and Integrated*

1. On a five-day trip, the Jansens drove 1400 miles. What was the
(50) average number of miles the Jansens drove on each of the five days?

2. **Estimate** Round both 634 and 186 to the nearest hundred to estimate
(62) their product before multiplying.

3. a. $\frac{1}{10} = \frac{\square}{100}$
(71, 79)

    b. What percent equals the fraction $\frac{1}{10}$?

4. The weight of an object on the moon is about $\frac{1}{6}$ of the weight of the
(46) same object on Earth. A person who weighs 108 pounds on Earth
would weigh about how many pounds on the moon?

*5. **Connect** Name the total number of shaded circles as an
(113) improper fraction and as a mixed number.
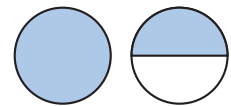
6. An inch is about 2.5 centimeters.
(53, 72)

    a. What is the perimeter of this square in inches?
    In centimeters?

    b. What is the area of this square in square inches?
    In square centimeters?
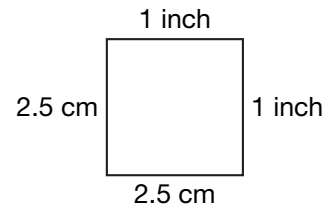
*7. What fraction of a year is 3 months? What percent of a year is
(81, 107) 3 months?

8. a. Name the shape at right.
(83)

    b. How many faces does the shape have?
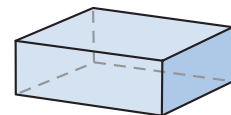
*9. The denominators of $\frac{1}{6}$ and $\frac{1}{4}$ are 6 and 4. What is the least common
(112) multiple (LCM) of the denominators?

10. **Connect** To what mixed number is the arrow pointing?
(38, 81)
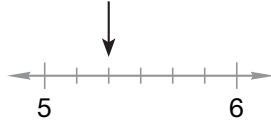
11. $4.239 + 25 + 6.79 + 12.5$
(99)

*12. $6.875 - (4 - 3.75)$
(24, 102)

*13. $\begin{array}{r} 3.7 \\ \times\ 0.8 \\ \hline \end{array}$
(109)

*14. $\begin{array}{r} 0.125 \\ \times\ \ 100 \\ \hline \end{array}$
(111)

*15. $\begin{array}{r} 0.32 \\ \times\ 0.04 \\ \hline \end{array}$
(110)

16. $\dfrac{408}{17}$
(94)

17. $27\overline{)705}$
(94)

18. $5\overline{)\$17.70}$
(26)

19. $\begin{array}{r} 3\frac{7}{10} \\ +\ 4 \\ \hline \end{array}$
(43)

20. $\begin{array}{r} 5\frac{5}{8} \\ +\ \frac{1}{8} \\ \hline \end{array}$
(81)

21. $\begin{array}{r} 7 \\ -\ 4\frac{3}{10} \\ \hline \end{array}$
(63)

22. $\frac{5}{6}$ of 4
(86)

*23. $\frac{3}{8} \times \frac{1}{2}$
(76)

*24. $\frac{3}{8} \div \frac{1}{2}$
(96)

*25. Josette spent $\frac{1}{6}$ of an hour walking to school and $\frac{1}{4}$ of an hour walking
(79) home from school. How many minutes did Josette spend walking to and from school? What fraction of an hour did Josette spend walking to and from school? (*Hint:* Write fractions equal to $\frac{1}{6}$ and $\frac{1}{4}$ that have denominators of 12. Then add the fractions.)

*26. **a.** What is the volume of a chest of drawers with the
(103)   dimensions shown?

b. What is the area of the top of the chest?

c. What is the perimeter of the top of the chest?

3 ft
2 ft
5 ft
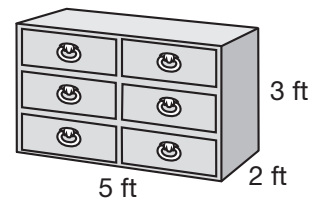
27. ✏️ **Explain** Tiana mailed two packages at the post office. One
(43) package weighed $2\frac{1}{4}$ pounds, and the other weighed $3\frac{3}{4}$ pounds. The clerk told Tiana that the total weight of the packages was exactly 6 pounds. Was the clerk correct? Explain your answer.

*28. Lillian is planning a trip from San Diego to San Luis Obispo. The
(21, 108)   schedules for the trains she plans to take are printed below. Use this
information to answer parts **a–c.**

Station		#29		#48
San Diego	Dp	9:30 a.m.	Ar	7:50 p.m.
Anaheim	Ar	11:26 a.m.		5:51 p.m.
Los Angeles		12:30 p.m.		4:55 p.m.
Ventura		2:21 p.m.		2:39 p.m.
Santa Barbara		3:10 p.m.		1:40 p.m.
Solvang		4:05 p.m.		12:45 p.m.
San Luis Obispo		5:30 p.m.	Ar	11:10 a.m.
Paso Robles	Ar	6:20 p.m.	Dp	10:00 a.m.

a. The trip from San Diego to San Luis Obispo takes how long?

b. Train #48 departs Santa Barbara 15 minutes after it arrives. At what
time does the train depart from Santa Barbara?

c. **Multiple Choice** The distance between San Diego and San Luis
Obispo is about 320 miles. From departure to arrival, the train
travels about how many miles each hour?

A 30 miles   **B** 40 miles   **C** 50 miles   **D** 60 miles

*29. The girls' softball team held a fundraiser by selling calendars. Reyna
(49)   sold twice as many calendars as Mackenzie, and Cherise sold four
more calendars than Reyna. Mackenzie sold ten calendars. How many
calendars did Cherise sell?

*30. Use the table to solve parts **a** and **b.**
(84)

**Number of School Days per Year
(by country)**

Country	Number of School Days
China	251
Japan	243
Korea	220
United States	180

a. Find the median of the data.

b. Find the range of the data.

• Using Formulas

Power Up

**mental
math**

   a. Number Sense: Matthew spent $2\frac{1}{2}$ hours doing homework on Monday, $1\frac{1}{2}$ hours on Tuesday, and 2 hours on Wednesday. What was the average amount of time he spent per day on homework?

   b. Measurement: It took the turtle one minute to travel $2\frac{1}{4}$ feet. How many inches is $2\frac{1}{4}$ feet?

   c. Fractional Parts: $\frac{1}{8}$ of 24

   d. Fractional Parts: $\frac{3}{8}$ of 24

   e. Fractional Parts: $\frac{5}{8}$ of 24

   f. Powers/Roots: 4^3

   g. Calculation: 25% of 40, $+\ 2$, $\times\ 2$, $+\ 1$, $\div\ 5$, $\times\ 3$, $+\ 1$, $\div\ 8$, $-\ 2$

   h. Roman Numerals: Compare: MDXX $\bigcirc$ 1520

**problem
solving**

Choose an appropriate problem-solving strategy to solve this problem. Jamisha had 24 square tiles on her desk. She arranged them into a rectangle made up of one row of 24 tiles. Then she arranged them into a new rectangle made up of two rows of 12 tiles.

Draw two more rectangles Jamisha could make using all 24 tiles.

New Concept

Formulas describe processes for solving certain types of problems. Formulas often use letters and other symbols to show the relationship between various measures.

In the examples that follow, we use formulas to solve problems about perimeter, area, and volume.

Example 1

The Jacksons have added a dining room to a corner of their house. Mr. Jackson purchased crown molding that will be installed at the intersection of the walls and the ceiling of his dining room. Crown molding costs $5 per foot to install. What will be Mr. Jackson's cost for having the crown molding installed?
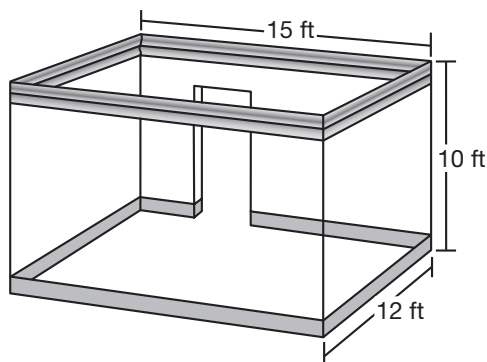
Crown molding is installed around the perimeter of the room. We can use the perimeter formula to determine the total length of crown molding and then multiply that length by $5 to find the cost of installation.

$$P = 2l + 2w$$
$$P = 2(15 \text{ ft}) + 2(12 \text{ ft})$$
$$P = 54 \text{ ft}$$

The perimeter is 54 ft, so the cost of the installed molding is $5 × 54 ft, which is **$270.**

Verify Why is the perimeter recorded in feet and not in square feet?

Example 2

Mrs. Jackson wants to buy carpet for the dining room floor. How many square feet of carpet are needed to cover the floor?
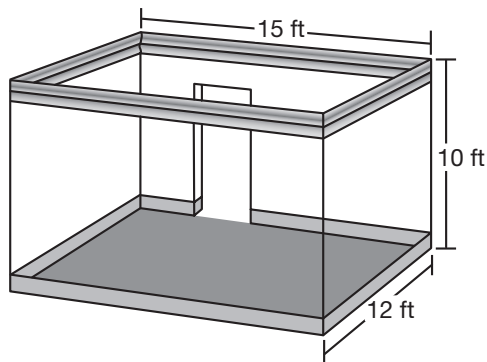
The carpet covers the floor area of the room, so we use the area formula to determine the amount of carpet needed.

$$A = l \times w$$
$$A = 15 \text{ ft} \times 12 \text{ ft}$$
$$A = 180 \text{ sq. ft}$$

Mrs. Jackson will need **180 sq. ft** of carpet to cover the floor.

(**Analyze**) The carpet Mrs. Jackson chose costs $5 per square foot. How much will the carpet cost?

Example 3

To heat and cool the new room, the Jacksons need to know the volume of the room. How many additional cubic feet of air need to be heated or cooled?
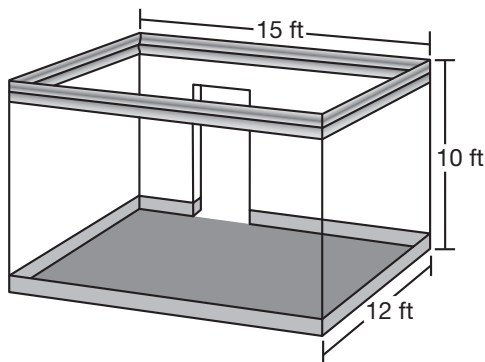
We use the volume formula to determine the amount of cubic feet added to the house.

$$V = l \times w \times h$$
$$V = 15 \text{ ft} \times 12 \text{ ft} \times 10 \text{ ft}$$
$$V = 1800 \text{ cu. ft}$$

The Jacksons have added **1800 cu. ft** of air to heat or cool.

(**Verify**) Why is the answer recorded in cubic feet and not in square feet?

Example 4

The Jacksons' son, Demont, has a trunk in his room for storing toys.

24 in.

30 in.

36 in.
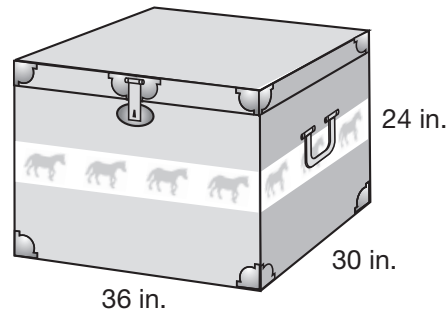
a. Mrs. Jackson plans to put a liner on the floor of the trunk. Choose a formula and use it to decide how much area the liner will cover.

b. Mrs. Jackson also plans to paste a border around the entire trunk. Choose a formula and use it to determine the minimum length of border she needs to buy.

a. The shape of the floor of the trunk is a rectangle. We use the area formula to find the area of the 36 in. by 30 in. rectangle.

$$A = l \times w$$

$$A = 36 \text{ in.} \times 30 \text{ in.}$$

$$A = \textbf{1080 sq. in.}$$

b. The border is pasted to the perimeter of the trunk, so we find the perimeter of a 36 in. by 30 in. rectangle.

$$P = 2l + 2w$$

$$P = 2(36 \text{ in.}) + 2(30 \text{ in.})$$

$$P = 132 \text{ in.}$$

Mrs. Jackson needs at least **132 in.** of border.

Lesson Practice

Refer to the diagrams in the examples of this lesson to help you answer problems **a** and **b.** For each practice problem, show the formula you can use.

a. The Jacksons want to cover one 12-foot long wall of the dining room with wallpaper. How many square feet will the wallpaper need to cover?

b. Calculate the storage capacity of Demont's toy box in cubic feet. (*Hint:* 30 inches is 2.5 feet.)

c. The diagram below is a top view of the Jackson house showing the outside walls. The dashes show the outside walls of the new dining room. Calculate the perimeter of the house.

15 ft   25 ft

12 ft

35 ft
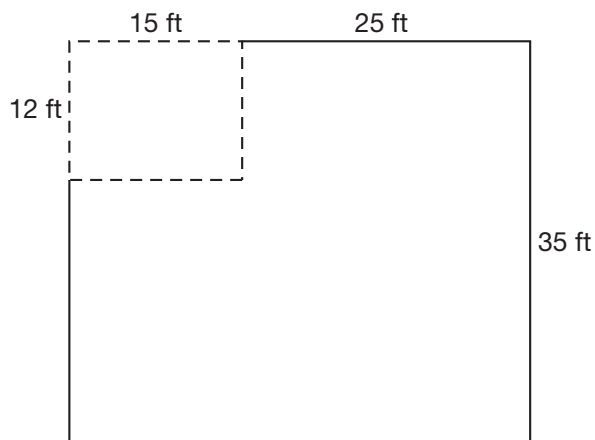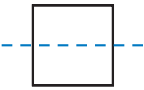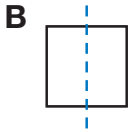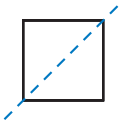
Written Practice   *Distributed and Integrated*

1. (Represent) Draw a circle and shade all but $\frac{1}{3}$ of it. What percent of the
(Inv. 3, 37) circle is shaded?

2. **Multiple Choice** Which of these units of length would probably be
(74) used to measure the length of a room?

  A inches   **B** feet

  C miles   **D** light-years

*3. **Multiple Choice** Which of these does *not* show a line of symmetry?
(105)

  A   **B**   **C**   **D**

*4. (Explain) Garcia's car can travel 28 miles on one gallon of gas. How
(21) far can his car travel on 16 gallons of gas? Explain why your answer is
reasonable.

5. Write $1\frac{3}{4}$ as an improper fraction.
(113)

6. (Explain) Is it possible for one friend to eat $\frac{1}{3}$ of a sandwich and for
(79, 81) another friend to eat $\frac{5}{6}$ of the same sandwich? Explain why or why not.

*7. The denominators of $\frac{3}{8}$ and $\frac{5}{6}$ are 8 and 6. What is the least common
(112) multiple (LCM) of the denominators?

8. Refer to this spinner to answer parts **a** and **b.**
(57)
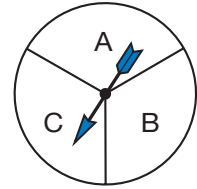
    a. What fraction names the probability that with one spin
the spinner will stop on sector A?

    b. What is the probability that with one spin the spinner
will stop on sector B?

9. *QS* is 6 cm. *RS* is 2 cm. *RT* is 6 cm. Find *QT.*
(61)

          Q   *R*   *S*   *T*

10. $45 + 16.7 + 8.29 + 4.325$
(99)

11. $4.2 - (3.2 - 1)$ ***12.** 0.75×0.05
(24, 99)   (110)

*13. 0.6×38 ***14.** 100×7.5
(109)   (111)

15. $24.36 \div 12   **16.** $4600 \div 5^2$
(92)   (78, 94)

17. $6\frac{9}{10} - \frac{1}{10}$ **1.** $5\frac{4}{9} + 3\frac{5}{9}$
(81)   (75)

1. $4 \div \frac{1}{8}$ **2.** $4 \times \frac{1}{8}$
(96)   (90)

*21 At a practice baseball game there were 18 players and 30 spectators.
(97) What was the ratio of players to spectators at the game?

*22   (**Connect**) What percent of the rectangle is shaded?
(107)

     What percent of the rectangle is not shaded?
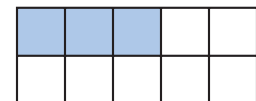
23   Write the reduced fraction equal to 60%.
(71, 90)

    Write the reduced fraction equal to 70%.

24. a. **Analyze** A loop of string can be arranged to form a
(53, 72) rectangle that is 12 inches long and 6 inches wide. If the
same loop of string is arranged to form a square, what
would be the length of each side of the square?

12 in.

6 in.
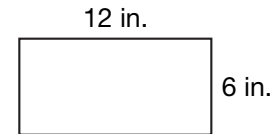
b. What is the area of the rectangle pictured in part **a?**

c. What is the area of the square described in part **a?**

*25. Find the percent equivalent to $\frac{1}{6}$ by multiplying 100% by $\frac{1}{6}$ and writing
(107) the answer as a mixed number with the fraction reduced.

26. **Explain** What is the result of doubling $7\frac{1}{2}$ and dividing the product
(49) by 3? Explain why your answer is reasonable.

27. In Duluth, Minnesota, the average January high temperature is 18°F.
(98) The average January low temperature is −1°F. How many degrees
greater is a temperature of 18°F than a temperature of −1°F?

*28. Each morning of a school day, Chelsea's alarm wakes her up at a
(28) quarter past six, and she leaves for school at a quarter to eight. What
mixed number represents the number of hours Chelsea spends on
those mornings getting ready for school?

*29. **Explain** The baseball cleats that Orin purchased online arrived
(104) in a shoe box. The box measured $11\frac{3}{8}$ in. by $8\frac{3}{4}$ in. by 4 in. Estimate
the volume of the box, and then explain why your estimate is
reasonable.

*30. Two squares form this hexagon. Refer to this figure for parts
(72) **a** and **b.**

3 cm
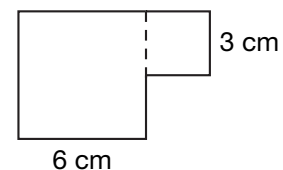
a. What is the area of each square?

6 cm

b. Combine the areas of the two squares to find the area
of the hexagon

Early Finishers

Real-World Connection

A community center is planning to build a tennis court. A regulation
tennis court has a length of 78 ft and a width of 36 ft. In addition, a
space of 12 ft is needed on each side of the court and a clearance of
21 ft is needed on each end of the court. Find the area of the entire
ground space needed for the tennis court. Be sure to show your work.

• Area, Part 2

Power Up

facts   Power Up K

mental math

a. Geometry: The sides of a square are 5 inches long. What is the perimeter of the square? What is the area of the square?

b. Geometry: Two angles of the triangle each measure 58°. The other angle measures 64°. What is the sum of the three angle measures?

c. Number Sense: Linda read 21 pages on Friday, 38 pages on Saturday, and 40 pages on Sunday. What was the average number of pages Linda read per day?

d. Percent: 25% of 80

e. Percent: 50% of 80

f. Percent: 75% of 80

g. Estimation: Suzie measured the length of the violin as $23\frac{1}{4}$ inches. Express this length as a mixed measure containing feet and inches.

h. Roman Numerals: Compare: 92 $\bigcirc$ LXXXII

problem solving

Choose an appropriate problem-solving strategy to solve this problem. Heather just found out that she won the poetry contest, and she is eager to spread the news among her friends and family. Heather told three people about her accomplishment. Then those three people each told two more people. Then each of those people told two more people. How many people other than Heather have received the news?

New Concept

Recall that we calculate the area of a rectangle by multiplying its length and width. In this lesson we will calculate the area of figures that can be divided into rectangles.

Example

Two rectangles are joined to form a hexagon. What is the area of the hexagon?

The hexagon can be divided into two rectangles. We find the area of each rectangle and then we add the areas to find the area of the hexagon.

Thinking Skill

Verify

How many sides does a hexagon have? Do all hexagons have congruent sides?

Area I	7 ft × 5 ft = 35 sq. ft
+ Area II	4 ft × 3 ft = 12 sq. ft
Combined area	**47 sq. ft**

Lesson Practice

Represent Copy each figure on your paper. Then find the area of each figure by dividing it into two rectangles and adding the areas of the parts.

Visit www. SaxonMath.com/ Int5Activities for an online activity.

a.

b.

c.

d.
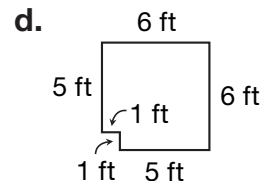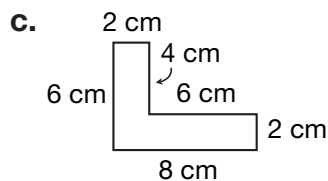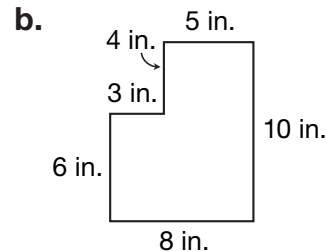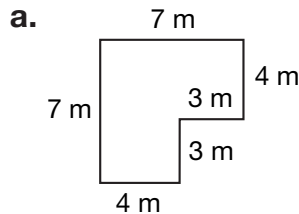
Written Practice

Distributed and Integrated

1. One length of string is 48 inches long. Another length of string is 24 feet long. Find the difference of those lengths.
(35, 74)

*2. **Connect** Name the total number of shaded circles below as an
(113) improper fraction and as a mixed number.
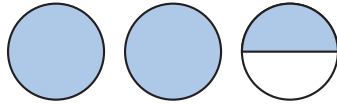
3. a. What fraction names the probability that with one spin,
(57) the spinner will stop on sector A?

   b. What is the probability that with one spin, the spinner
   will stop on sector B?

4. **Connect** To what mixed number is the arrow pointing?
(38)

7   8
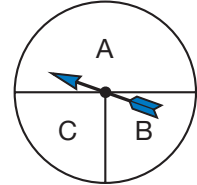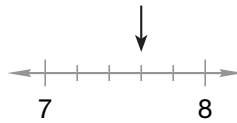
5. Lawrencia's first class of the afternoon begins $1\frac{1}{2}$ hours after 11:40 a.m.
(28) What time of the day does her first class of the afternoon begin?

6. Multiple Choice Which pair of fractions has the same
(Inv. 2) denominator?

   A $\frac{1}{3}, \frac{1}{4}$ **B** $\frac{4}{3}, \frac{4}{2}$ **C** $\frac{1}{4}, \frac{3}{4}$ **D** $\frac{5}{2}, \frac{5}{8}$

*7. The denominators of $\frac{2}{5}$ and $\frac{2}{3}$ are 5 and 3. Find the least common
(112) multiple (LCM) of the denominators.

*8. a. Estimate the perimeter of this rectangle.
(53, 104)

   b. Estimate the area of this rectangle.

3.98 m

2.96 m
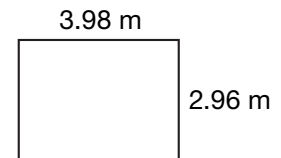
9. 42.98 + 50 + 23.5 + 0.025
(99)

*10. **Represent** How much greater than 5.18 is 6? Use words to write your
(68, 102) answer.

11.   0.375   ***12.**   0.14*   ***13.**   7.8
(111) × ____10   (110) × 0.06   (109) × __19

14. $2340 \div 30$
(54)

15. $18\overline{)2340}$
(94)

16. $7\overline{)8765}$
(26)

17. $\dfrac{5}{6} + 1\dfrac{5}{6}$
(91)

18. $7\dfrac{5}{8} - 7\dfrac{1}{8}$
(90)

*19. $\dfrac{4}{5} \times \dfrac{2}{3}$
(76)

*20. $\dfrac{4}{5} \div \dfrac{2}{3}$
(96)

21. $\dfrac{2}{5} = \dfrac{\square}{15}$
(79)

22. $\dfrac{2}{3} = \dfrac{\square}{15}$
(79)

23. In problems **21** and **22** you made fractions equal to $\frac{2}{5}$ and $\frac{2}{3}$ with
(91) denominators of 15. Add the fractions you made. Remember to convert the answer to a mixed number.

24. **a.** What is the perimeter of this regular pentagon?
(53, 105)
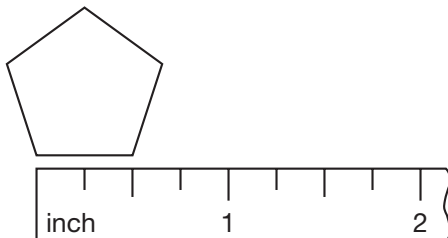
b. (**Justify**) Explain how you found your answer for part **a.**

c. A regular pentagon has how many lines of symmetry?

*25. **a.** What is the area of this hexagon?
(53, 115)

b. What is the perimeter of the hexagon?
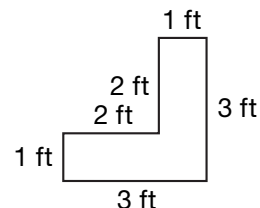
*26. What fraction of a square mile is a field that is $\frac{1}{2}$ mile long and $\frac{1}{4}$ mile
(76) wide?
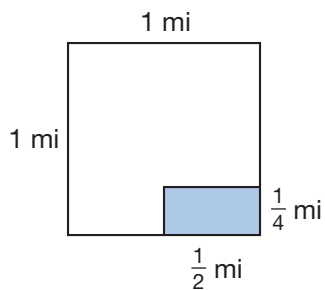
*27. Sandie says that multiplying $4\frac{1}{8}$ by 3 and then subtracting 1 gives an
(49) answer of $11\frac{3}{8}$. Explain how rounding can be used to decide if Sandie's
answer is reasonable.

*28. (Interpret) The table shows the average monthly temperatures during
(Inv. 6) autumn in Caribou, Maine. Display the data in a line graph.

**Average Monthly
Autumn Temperatures
Caribou, ME**

Month	Temperature (°F)
September	54
October	43
November	31

*29. **a.** Does this prism have parallel lines?
(89)
b. Does this prism have perpendicular lines?
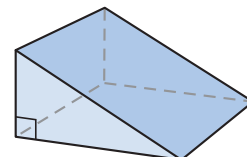
*30. Two squares form this hexagon. If the squares were
(53) separated, their perimeters would be 12 cm and 24 cm,
respectively. However, the perimeter of the hexagon is not
the sum of the perimeters of the squares because all of
the sides of the small and large squares are not part of the
perimeter of the hexagon. Copy the hexagon on your paper,
and show the length of each of the six sides. What is the
perimeter of the hexagon?

3 cm

6 cm
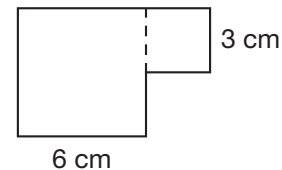
**Early
Finishers**

*Real-World
Connection*

Mr. Rio plans to cover his backyard with grass. The
diagram below shows the dimensions of his backyard.
How many square yards of grass are needed?

20 yd

9 yd

15 yd

14 yd

3 yd

3 yd
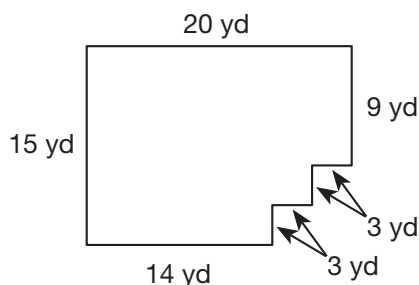
• Finding Common Denominators to Add, Subtract, and Compare Fractions

Power Up

facts   Power Up K

mental math

 a. Geometry: A rectangle is 6 inches long and 4 inches wide. What is its perimeter? What is its area?

 b. Time: How many seconds is two and a half minutes?

 c. Percent: What is 10% of $300? … 10% of $30? … 10% of $3?

 d. Number Sense: $2 - \frac{3}{5}$

 e. Fractional Parts: $\frac{1}{2}$ of 81

 f. Probability: Jill's schoolbag contains 2 red pens, 4 black pens, 1 blue pen, and 1 green pen. If Jill selects one pen without looking, what is the probability it will be a black pen? Express this number as a percent.

 g. Calculation: $\sqrt{16}$, $\times 5$, $- 6$, $\div 7$, $+ 8$, $\times 9$, $\div 10$

 h. Roman Numerals: Compare: CCCIV $\bigcirc$ 340

problem solving   Choose an appropriate problem-solving strategy to solve this problem. The local newspaper sells advertising for $20 per column inch per day. An ad that is 2 columns wide and 4 inches long is 8 column inches ($2 \times 4 = 8$) and costs $160 each day ($8 \times 20). What would be the total cost of running a 3-column by 8-inch ad for two days?

New Concept

Thinking Skill

Justify

Why do fractions need common denominators? What does it mean to add fractions with common denominators?

The fractions $\frac{1}{4}$ and $\frac{3}{4}$ have common denominators. The fractions $\frac{1}{2}$ and $\frac{1}{4}$ do not have common denominators. Fractions have common denominators if their denominators are equal.

Common denominators	Different denominators
$\frac{1}{4} \longleftrightarrow \frac{3}{4}$	$\frac{1}{2} \longleftrightarrow \frac{1}{4}$

To compare, add, or subtract fractions that have different denominators, we first change the name of one or more of the fractions so that they have common denominators. The least common multiple (LCM) of the denominators is the least common denominator of the fractions. The denominators of $\frac{1}{2}$ and $\frac{1}{4}$ are 2 and 4. The LCM of 2 and 4 is 4, so the least common denominator for halves and fourths is 4.

Example 1

In one of Katie's cookbooks, a recipe for salsa calls for $\frac{3}{4}$ cup of chopped fresh cilantro. A salsa recipe given to Katie by a friend calls for $\frac{7}{8}$ cup of chopped fresh cilantro. Which recipe calls for more cilantro?

Rewriting fractions with common denominators can help us compare fractions. The denominators are 4 and 8. We change fourths to eighths by multiplying by $\frac{2}{2}$.

$$\frac{3}{4} \times \frac{2}{2} = \frac{6}{8}$$

We see that $\frac{6}{8}$ is less than $\frac{7}{8}$. We can also express the comparison with a less than sign:

$$\frac{3}{4} < \frac{7}{8}$$

We find that the **recipe from Katie's friend** calls for more cilantro than the cookbook recipe.

Example 2

Add: $\frac{1}{2} + \frac{1}{4}$

Since $\frac{1}{2}$ and $\frac{1}{4}$ have different denominators, we change the name of $\frac{1}{2}$ so that both fractions have a denominator of 4. We change $\frac{1}{2}$ to fourths by multiplying by $\frac{2}{2}$, which gives us $\frac{2}{4}$.

$$\frac{1}{2} \times \frac{2}{2} = \frac{2}{4}$$

Then we add $\frac{2}{4}$ and $\frac{1}{4}$ to get $\frac{3}{4}$.

$$\frac{2}{4} + \frac{1}{4} = \mathbf{\frac{3}{4}}$$

Example 3

Subtract:

$$3\frac{1}{2}$$
$$-1\frac{1}{6}$$

We work with the fraction part of each mixed number first. The denominators are 2 and 6. We can change halves to sixths. We multiply $\frac{1}{2}$ by $\frac{3}{3}$ and get $\frac{3}{6}$.

$$\frac{1}{2} \times \frac{3}{3} = \frac{3}{6}$$

Then we subtract and reduce the answer.

$$3\frac{3}{6}$$
$$-1\frac{1}{6}$$
$$2\frac{2}{6} = \mathbf{2\frac{1}{3}}$$

Example 4

Add: $\frac{1}{3} + \frac{1}{2}$

For this problem we need to rename *both* fractions. The denominators are 3 and 2. The LCM of 3 and 2 is 6, so the least common denominator for thirds and halves is sixths. We rename the fractions and then add.

$$\frac{1}{3} \times \frac{2}{2} = \frac{2}{6}$$

$$+\frac{1}{2} \times \frac{3}{3} = \frac{3}{6}$$

$$\mathbf{\frac{5}{6}}$$

Lesson Practice

For problems **a–c,** find a common denominator and compare.

a. Bart spent $\frac{7}{12}$ of an hour on math and $\frac{2}{3}$ of an hour reading. Compare $\frac{7}{12}$ and $\frac{2}{3}$ to find whether Bart spent more time on math or reading.

b. Copy these fractions and replace the circle with the correct comparison symbol. $\frac{2}{5} \bigcirc \frac{1}{3}$

c. The twins took turns carrying the tent up the mountain. Larry carried the tent $\frac{5}{10}$ of the distance, and Barry carried it $\frac{2}{4}$ of the distance. Who carried the tent farther?

For problems **d–q,** find each sum or difference. As you work the problems, follow these steps:

- Find the common denominator.
- Rename one or both fractions.
- Add or subtract the fractions.
- Reduce the answer when possible.

d. $\frac{1}{2} + \frac{1}{8}$ **e.** $\frac{1}{2} - \frac{1}{4}$ **f.** $\frac{3}{4} + \frac{1}{8}$

g. $\frac{2}{3} - \frac{1}{9}$ **h.** $\frac{1}{3} + \frac{1}{4}$ **i.** $\frac{1}{2} - \frac{1}{3}$

j. $3\frac{1}{4}$ **k.** $2\frac{1}{8}$ **l.** $3\frac{1}{2}$ **m.** $2\frac{3}{4}$
$+ 2\frac{1}{2}$ $+ 5\frac{1}{2}$ $- 1\frac{1}{6}$ $- 2\frac{1}{2}$

n. $5\frac{5}{8}$ **o.** $3\frac{1}{2}$ **p.** $4\frac{3}{4}$ **q.** $4\frac{1}{2}$
$+ 1\frac{1}{4}$ $+ 1\frac{1}{3}$ $- 1\frac{2}{3}$ $- 1\frac{1}{5}$

Written Practice *Distributed and Integrated*

*1. **Represent** Draw a circle. Shade all but $\frac{1}{6}$ of it. What percent of the
(37, 107) circle is shaded?

2. In 1875 Bret Harte wrote a story about the California Gold Rush of
(35) 1849. How many years after the Gold Rush did he write the story?

*3. **a.** What is the chance of the spinner stopping on 4 with one spin?
(57, 107)

   b. What is the probability that with one spin the spinner will stop on a number less than 4?

*4. **Multiple Choice** Which of these does *not* show a line of symmetry?
(105)

A   **B**   **C**   **D**
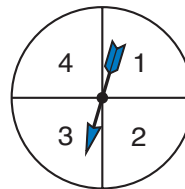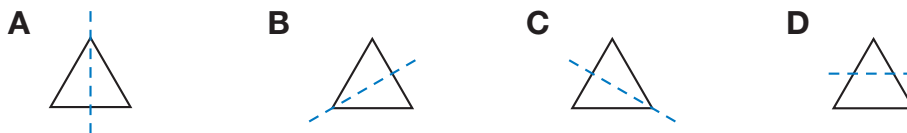
*5. Compare these fractions. First write the fractions with common denominators.
(116)

$$\frac{2}{3} \bigcirc \frac{5}{6}$$

*6. **Connect** Name the total number of shaded circles as an improper fraction and as a mixed number.
(113)
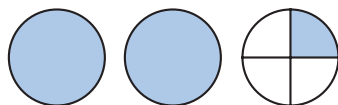
7. Alberto counted 100 cars and 60 trucks driving by the school. What was the ratio of trucks to cars that Alberto counted driving by the school?
(97)

*8. **a.** What is the perimeter of this square?
(53, 73, 109)

   b. What is the area of this square?

0.5 cm
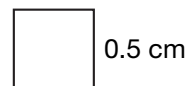
9. $\overline{AC}$ is 70 mm. $\overline{BC}$ is 40 mm. $\overline{BD}$ is 60 mm. Find the length of $\overline{AD}$.
(61)

A   B   C   D

*10. $\dfrac{1}{4} + \dfrac{1}{8}$
(116)

*11. $\dfrac{3}{4} - \dfrac{1}{2}$
(116)

*12. $\dfrac{7}{8} - \dfrac{3}{4}$
(116)

*13. $2\dfrac{5}{8}$
(116) $-1\dfrac{1}{2}$

*14. $3\dfrac{1}{2}$
(116) $-2\dfrac{1}{8}$

*15. $5\dfrac{1}{6}$
(116) $+1\dfrac{1}{3}$

16. $\frac{3}{5} \times 3$
(86, 91)

17. $3 \div \frac{3}{5}$
(96)

***18.** 6.5×100
(111)

***19.** 4.6×80
(109)

***20.** 0.18×0.4
(110)

21. $10\overline{)\$13.20}$
(54)

22. $12\overline{)\$13.20}$
(92)

23. $1470 \div 42$
(94)

24. Which angle in quadrilateral *ABCD* is an obtuse angle?
(31, 61)
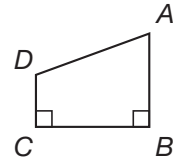
***25.** Add these fractions. First rename the fractions so that they have a
(116) common denominator of 12.

$$\frac{1}{4} + \frac{2}{3}$$

***26.** What is the area of this figure?
(115)
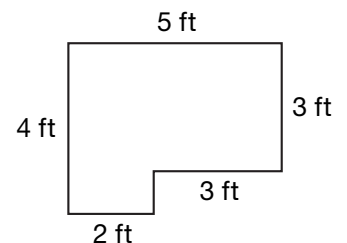
27. ✏ (**Explain**) Is it possible to arrange exactly 85 chairs in 12 rows and
(92) have the same number of chairs in each row? Explain why or why not.

28. ✏ (**Explain**) The capacity of the fuel tank on Jim's car is 12.3 gallons,
(51) and Jim can travel an average of 29 miles for every gallon of fuel
his car uses. What is a reasonable estimate of the distance Jim
can travel with one full tank of fuel? Explain why your estimate is
reasonable.

***29.** ✏ (**Explain**) These fractions do not add to the same sum.
(116)

$$\frac{2}{3} + \frac{3}{4} \qquad \frac{3}{8} + \frac{2}{5}$$

Which sum is greater? Explain how you can compare each addend to $\frac{1}{2}$ to find the
answer.

*30. Tim plans to take the train from Fort Collins, where he attends college,
(108) to Union Station in Denver. From Union Station he will take a taxi to a
job interview, meet a friend for dinner, and then return to Fort Collins
at night. Use this information and the train schedule below to solve
parts **a–c.**

Cheyenne • Fort Collins • Denver

6		Connecting Train Number		6
11:30 a.m.	Dp	Cheyenne, WY	Ar	12:30 a.m.
12:30 p.m.		Fort Collins, CO	↑	11:40 p.m.
12:40 p.m.		Loveland, CO		11:30 p.m.
1:00 p.m.		Longmont, CO		11:10 p.m.
1:35 p.m.	↓	Boulder, CO	Ar	10:35 p.m.
2:05 p.m.	Ar	Denver, CO	Dp	9:00 p.m.

a. Tim plans to study on the ride south to Denver. How long will he
have for studying between the departure from Fort Collins and the
arrival in Denver?

b. While he's in Denver, Tim plans to have dinner with a friend.
The drive from the train station to the restaurant takes about
20 minutes. Tim wants to be back at the train station an hour
before its departure from Denver. By what time should Tim and
his friend leave the restaurant?

c. **Explain** If the college campus is a 5-minute walk from the
Fort Collins train station, can Tim be back on campus by midnight?
Explain your answer.

Early Finishers

Real-World Connection

Hakib skied three trails last month. The first trail was $1\frac{3}{4}$ miles long,
the second trail was $3\frac{1}{2}$ miles long, and the third trail was $4\frac{7}{8}$ miles long.

a. What is the difference between the longest trail and the shortest
trail?

b. Altogether, how many miles did Hakib ski last month?

• Dividing a Decimal Number by a Whole Number

Power Up

facts   Power Up K

mental math

a. **Estimation:** Using compatible numbers, estimate the per gallon price for a 29.6 gallon tank that cost $64 to fill with gas.

b. **Time:** The Richardsons left their house at 7:45 a.m. and returned home at 4:15 p.m. How long were they gone?

c. **Time:** The space shuttle makes one orbit of the earth in about 90 minutes. About how long does it take the space shuttle to make 3 orbits?

d. **Percent:** The board game is regularly priced at $20. It is on sale for 10% off. What is the sale price?

e. **Money:** Which coin has a value equal to $\frac{1}{8}$ of $2?

f. **Measurement:** To make lemonade, Yoshi used 10 cups of water. How many pints of water is 10 cups?

g. **Calculation:** $\frac{1}{5}$ of 20, $\times$ 4, $-$ 4, $\div$ 4, $+$ 4, $\times$ 4

h. **Roman Numerals:** Compare: 110 $\bigcirc$ XC

problem solving   Choose an appropriate problem-solving strategy to solve this problem. Find the next three numbers in this sequence:

1, 1, 2, 3, 5, 8, 13, ____, ____, ____, …

Dividing a decimal number by a whole number is like dividing money by a whole number. The decimal point in the quotient is directly above the decimal point inside the division box. In the chart below, "÷ by whole (*W*)" means "division by a whole number." The memory cue "up" reminds us where to place the decimal in the quotient. (We will later learn a different rule for dividing by a decimal number.)

Thinking Skill

Generalize

How is dividing a money amount by a whole number the same as dividing two whole numbers? How is it different?

Decimals Chart

Operation	+ or −	×	÷ by whole (*W*)
Memory cue	line up . ± . ——— .	×; then count ._ × ._ ——— ._ _	up . $W)\overline{.}$

You may need to...
- Place a decimal point on the end of whole numbers.
- Fill each empty place with a zero.

We sometimes need to use one or more zeros as placeholders when dividing decimal numbers. Here we show this using money.

Suppose $0.12 is shared equally by 3 people. The division is shown below. Notice that the decimal point in the quotient is directly above the decimal point in the dividend. We fill each empty place with a zero and see that each person will receive $0.04.

$$\begin{array}{r} \$\ .\ 4 \\ 3)\overline{\$0.12} \\ 12 \\ \hline 0 \end{array} \longrightarrow \begin{array}{r} \$0.04 \\ 3)\overline{\$0.12} \\ 12 \\ \hline 0 \end{array} \text{ decimal point "up"}$$

Example 1

For an art project, Corbin must cut a length of ribbon in half. The ribbon is 4.8 meters long. If he cuts the ribbon correctly, how long will each length of ribbon be?

We are dividing 4.8 meters by 2, which is a whole number. We recall the memory cue "up" and place the decimal point in the answer directly above the decimal point inside the division box. Then we divide. Each length of ribbon will be **2.4 meters.**

$$\begin{array}{r} 2.4 \\ 2)\overline{4.8} \\ 4 \\ \hline 0\ 8 \\ 8 \\ \hline 0 \end{array}$$

Example 2

Divide: $3\overline{)0.42}$

We place the decimal point in the answer "straight up." Then we divide.

$$\begin{array}{r} 0.14 \\ 3\overline{)0.42} \\ \underline{3} \\ 12 \\ \underline{12} \\ 0 \end{array}$$

Example 3

Divide: $0.15 \div 3$

We rewrite the problem using a division box. The decimal point in the answer is "straight up." We divide and remember to fill empty places with zeros.

$$\begin{array}{r} 0.05 \\ 3\overline{)0.15} \\ \underline{15} \\ 0 \end{array}$$

Generalize How is dividing a decimal by a whole number similar to dividing a money amount by a whole number? How is it different?

Example 4

Divide: $0.0024 \div 3$

We rewrite the problem using a division box. The decimal point in the answer is "straight up." We divide and remember to fill empty places with zeros.

$$\begin{array}{r} 0.0008 \\ 3\overline{)0.0024} \end{array}$$

Lesson Practice   Divide:

a. $4\overline{)0.52}$ **b.** $6\overline{)3.6}$ **c.** $0.85 \div 5$

d. $5\overline{)7.5}$ **e.** $5\overline{)0.65}$ **f.** $2.1 \div 3$

g. $4\overline{)0.16}$ **h.** $0.35 \div 7$ **i.** $5\overline{)0.0025}$

j. $0.08 \div 4$ **k.** $6\overline{)0.24}$ **l.** $0.0144 \div 3$

m. **Estimate** A gallon is about 3.78 liters. About how many liters is half a gallon?

Written Practice

Distributed and Integrated

*1. **Represent** Write the following sentence using digits and symbols:
(6, 116)

The sum of one sixth and one third is one half.

*2. **(Analyze)** Gilbert scored half of his team's points. Socorro scored
(49) 8 fewer points than Gilbert. The team scored 36 points. How many
points did Socorro score?

3. In the northern hemisphere, the first day of winter is December 21
(28) or 22. The first day of summer is 6 months later. The first day of
summer can be on what two dates?

4. a. What are all the possible outcomes when the spinner is
(57, 107)   turned?
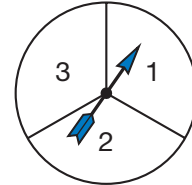
    b. What is the probability that with one spin the spinner will
stop on a number greater than one?

    c. What is the chance of landing on a three with one spin?

5. **(Connect)** Name the shaded portion of this rectangle as a
(71) fraction, as a decimal, and as a percent.
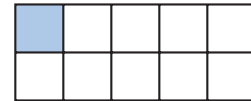
6. If each side of a regular octagon is 6 inches long, then the perimeter of
(32, 53) the octagon is how many feet? What formula could you use?

*7. **(Represent)** Name the total number of shaded circles as
(113) an improper fraction and as a mixed number.
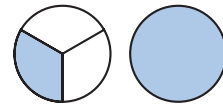
8. What is the largest four-digit odd number that uses the digits 7, 8, 9,
(2) and 0 once each?

*9. Refer to rectangle *ABCD* to answer problems **a–c.** In the
(53, 72) rectangle, *AB* is 3 cm and *BC* is 4 cm.
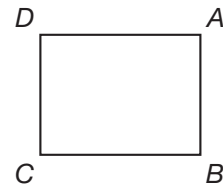
    a. Which segment is parallel to $\overline{AB}$?

    b. What is the perimeter of the rectangle?

    c. What is the area of the rectangle?

*10. L'Shawn's hallway locker measures 12 inches wide by 12 inches deep
(103) by 5 feet tall. What is the volume of the locker in cubic feet?

11. $\overline{KL}$ is 56 mm. $\overline{LM}$ is half of $\overline{KL}$. $\overline{MN}$ is half of $\overline{LM}$. Find $\overline{KN}$.
(61)

$$\underset{K}{\bullet} \quad\quad\quad\quad \underset{L}{\bullet} \quad\quad \underset{M}{\bullet} \quad \underset{N}{\bullet}$$
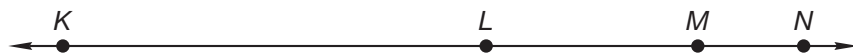
* 12. $16 + 3.17 + 49 + 1.125$
(99)

* 13. How much greater is 3.42 than 1.242?
(102)

* 14. 4.3×100　　　　　　　　*** 15.** 6.4×3.7
(111)　　　　　　　　　　　　　　　(109)

* 16. 0.36×0.04　　　　　　　*** 17.** $2\overline{)3.6}$
(110)　　　　　　　　　　　　　　　(117)

* 18. $7\overline{)0.0049}$　　　　　　　*** 19.** 1.35×90
(117)　　　　　　　　　　　　　　　(109)

* 20. $2\frac{1}{8}$　　　*** 21.** $\frac{1}{3}$　　　*** 22.** $\frac{7}{10}$　　　*** 23.** $3\frac{9}{10}$
(116)　　　　　(116)　　　　　(116)　　　　　(116)

$+ 1\frac{3}{4}$　　　　$+ \frac{1}{6}$　　　　$- \frac{1}{2}$　　　　$- \frac{1}{5}$

24. $4 \times \frac{3}{2}$　　　　**25.** $\frac{3}{4} \div \frac{1}{4}$　　　　**26.** Reduce: $\frac{18}{144}$
(86, 91)　　　　　　(96)　　　　　　　　　(90)

* 27. Find the sum of $3\frac{1}{5}$ and $2\frac{1}{2}$ by first rewriting the fractions with 10 as the
(116)　common denominator.

28. To finish covering the floor of a room, Abby needed a rectangular piece
(72, 76)　of floor tile 6 inches long and 3 inches wide.

6 in.

3 in.　$\frac{1}{4}$ ft

$\frac{1}{2}$ ft
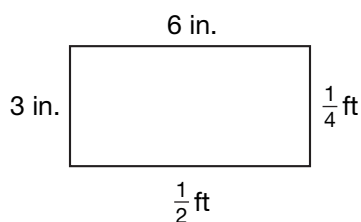
a. What is the area of this rectangle in square inches?

b. What is the area of the rectangle in square feet?

*29. A 2-by-2-inch square is joined with a 5-by-5-inch square
(53, 115) to form a hexagon. Refer to the figure to answer parts
a and **b.**
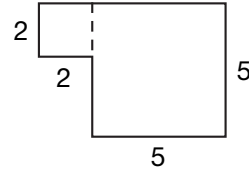
2 ⌐‾‾‾‾‾‾‾‾
2 │ 5
5

   a. What is the area of the hexagon?

   b. Copy the hexagon and show the lengths of all six sides. Then find
   the perimeter of the hexagon.

30. Mahdi is a jewelry designer. She has three irregular 10-karat gold
(49) nuggets. The weights of the nuggets are $28\frac{1}{3}$ grams, $56\frac{2}{3}$ grams,
and 85 grams. What is the total weight in grams of the nuggets?

Early Finishers

Real-World Connection

Mount Vesuvius is an active volcano located to the east of Naples, Italy.
To reach the top of the volcano, visitors must climb to an elevation
of 4202.76 feet. A group of hikers start their climbing from sea level
(elevation 0) and want to climb Mount Vesuvius in three days.

   a. If they want to gain the same amount of elevation each day, how
   many feet would this be?

   b. What if they climb down the mountain in two days? How many feet
   do they climb down per day

• More on Dividing Decimal Numbers

Power Up

mental math

a. **Number Sense:** 32×10

b. **Number Sense:** 16×20

c. **Number Sense:** 8×40

d. **Powers/Roots:** 5^3

e. **Measurement:** Two tables are each 48 inches long. If the tables are placed end to end, how many feet long is the resulting table?

f. **Time:** How many years is $\frac{1}{4}$ of a century?

g. **Number Sense:** $25 - 12\frac{1}{2}$

h. **Calculation:** $6^2 - 8, \div 7, \times 2, + 10, \div 2, \div 3$

problem solving

Choose an appropriate problem-solving strategy to solve this problem. How many 1-inch cubes would be needed to build a rectangular solid 5 inches long, 4 inches wide, and 3 inches high?

3 in.

4 in.

5 in.
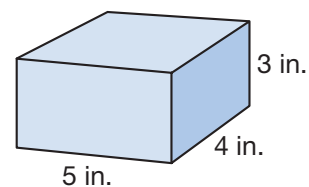
New Concept

We usually do not write remainders with decimal division problems. The procedure we will follow for now is to continue dividing until the "remainder" is zero. In order to continue the division, we may need to attach extra zeros to the decimal number that is being divided. **Remember that attaching extra zeros to the back of a decimal number does not change the value of the number.**

Example 1

Divide: 0.6 ÷ 5

The first number goes inside the division box. The decimal point is straight up. As we divide, we attach a zero and continue dividing.

$$
\begin{array}{r}
0.12 \\
5\overline{)0.60} \\
5 \\
\overline{10} \\
10 \\
\overline{0}
\end{array}
$$

Justify Why is 60 hundredths equal to 6 tenths?

Example 2

Divide: 0.3 ÷ 4

As we divide, we attach zeros and continue dividing. We fill each empty place in the quotient with a zero.

$$
\begin{array}{r}
0.075 \\
4\overline{)0.300} \\
28 \\
\overline{20} \\
20 \\
\overline{0}
\end{array}
$$

Verify Demonstrate how to check the answer.

Example 3

Divide: 3.4 ÷ 10

As we divide, we attach a zero to 3.4 and continue dividing. Notice that the same digits appear in the quotient and dividend, but in different places.

$$
\begin{array}{r}
0.34 \\
10\overline{)3.40} \\
3\,0 \\
\overline{40} \\
40 \\
\overline{0}
\end{array}
$$

Thinking Skill

Generalize

How is dividing by 10 similar to multiplying by 10? How is it different?

When we divide a number by 10, we find that the answer has the same digits, but the digits have shifted one place to the right.

$$
\begin{array}{cc}
3\,4. & .3\,4 \\
\uparrow\,\uparrow & \uparrow\,\uparrow \\
10\overline{)340.} & 10\overline{)3.40}
\end{array}
$$

We can use this pattern to find the answer to a decimal division problem when the divisor is 10. The shortcut is very similar to the method we use when multiplying a decimal number by 10. In both cases it is the digits that are shifting places.

However, we can make the digits appear to shift places by shifting the decimal point instead. To divide by 10, we shift the decimal point one place to the left.

$$3.4 \div 10 = .34$$

Dividing by 100 is like dividing by 10 twice. When we divide by 100, we shift the decimal point two places to the left. When we divide by 1000, we shift the decimal point three places to the left. We shift the decimal point the same number of places as there are zeros in the number we are dividing by (10, 100, or 1000). We can remember which way to shift the decimal point if we keep in mind that dividing a number into 10, 100, or 1000 parts produces **smaller** numbers. As a decimal point moves to the left, the value of the number becomes smaller and smaller.

Example 4

Mentally divide 3.5 by 100.

When we divide by 10, 100, or 1000, we can find the answer mentally without performing the division algorithm. To divide by 100, we shift the decimal point two places. We know that the answer will be less than 3.5, so we remember to shift the decimal point to the left. We fill the empty place with a zero.

$$3.5 \div 100 = \textbf{0.035}$$

Connect Explain how to mentally divide 3.5 by 1000.

Lesson Practice Divide:

a. $0.6 \div 4$ **b.** $0.12 \div 5$ **c.** $0.1 \div 4$

d. $0.1 \div 2$ **e.** $0.4 \div 5$ **f.** $1.4 \div 8$

g. $0.5 \div 4$ **h.** $0.6 \div 8$ **i.** $0.3 \div 4$

Mentally perform the following divisions:

j. $2.5 \div 10$ **k.** $32.4 \div 10$ **l.** $2.5 \div 100$

m. $32.4 \div 100$ **n.** $2.5 \div 1000$ **o.** $32.4 \div 1000$

p. $12 \div 10$ **q.** $12 \div 100$ **r.** $12 \div 1000$

Written Practice *Distributed and Integrated*

＊1. Multiple Choice Which of these shows two parallel line segments that are *not* horizontal?
(31)

A   B   C   D
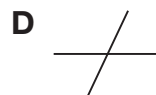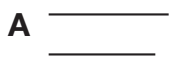
2. Byron estimated the product of $6\frac{1}{10}$ and $4\frac{7}{8}$ by first rounding each factor
(101) to the nearest whole number. What was his estimate ?

3. How many 12¢ pencils can V'Nessa buy with one dollar?
(21, 22)

4. **Multiple Choice** Which of these figures does *not* show a line of
(105) symmetry?

A **B** **C** **D**
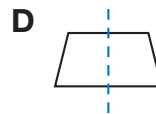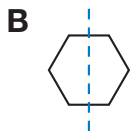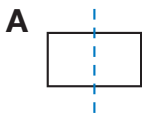
*5. The first roll of the bowling ball knocked down 3 of the 10 pins. What
(107) percent of the pins were still standing?

6. a. Write the fraction equal to 4%.
(71, 90)

b. Write the fraction equal to 5%.

7. (**Represent**) Name the total number of shaded circles as an improper
(113) fraction and as a mixed number.
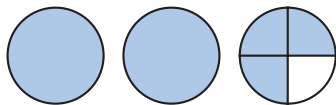
*8. Rihanna has been asked to divide $1\frac{3}{8}$ cups of wheat flour into two
(76, 113) equal amounts. She knows that the improper fraction $\frac{11}{8}$ can be
used to represent $1\frac{3}{8}$, and she knows that dividing by 2 is the same
as multiplying by $\frac{1}{2}$. How many cups of flour will each of the equal
amounts represent?

9. A stop sign has the shape of an 8-sided polygon. Name a polygon that
(105) has 8 sides. Does a stop sign have rotational symmetry?

*10. Arrange these numbers in order from least to greatest: $\frac{5}{3}, \frac{5}{6}, \frac{5}{5}$
(116)

*11. Neil worked on a task for $1\frac{1}{3}$ hours before taking a break. The task
(116) takes $2\frac{3}{4}$ hours to complete. After Neil begins working again, how long
will it take him to complete the task?

* 12. The perimeter of this square is 1.2 meters.
(53, 72)

    a. How long is each side of this square?

    b. What is the area of this square?
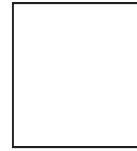
13. $49.35 + 25 + 3.7$
(99)

14. Compare: $\sqrt{81} + \sqrt{100} \bigcirc 9^2 + 10^2$
(78, 89)

15. (**Represent**) Subtract 1.234 from 2. Use words to write the answer.
(68, 102)

* 16. $0.0125 \div 5$ *** 17.** 4.2×100 **18.** 0.5×0.17
(117)   *(111)*   *(110)*

* 19. $0.6 \div 4$ *** 20.** $0.6 \div 10$ *** 21.** $4\overline{)1.8}$
(118)   *(118)*   *(118)*

* 22. $3\frac{1}{9}$ *** 23.** $\frac{1}{3}$ *** 24.** $\frac{7}{8}$ *** 25.** $4\frac{1}{2}$
(116)   *(116)*   *(116)*   *(116)*

$+ \ \frac{1}{3}$ $+ \ \frac{5}{6}$ $- \ \frac{1}{4}$ $- \ 1\frac{3}{10}$

26. $6 \times \frac{2}{3}$ **27.** $6 \div \frac{2}{3}$
(86, 91)   *(96)*

* 28. Divide mentally:
(118)

    a. $3.5 \div 100$ **b.** $87.5 \div 10$

* 29. A 2 cm by 3 cm rectangle is joined to a 4 cm by 6 cm
(53, 115) rectangle to form this hexagon. Refer to the figure to
answer problems **a** and **b.**
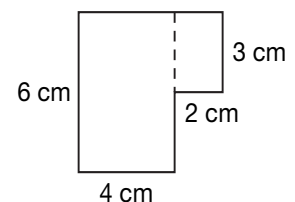
    a. What is the area of the hexagon?

    b. Copy the hexagon and show the lengths of all six
      sides.
      Then find the perimeter of the hexagon.

30. ✏️ (**Explain**) On his way home from work, Carter purchased 3 gallons
(49) of milk for $2.19 per gallon and 2 loaves of bread for $1.69 per loaf.
What is a reasonable estimate of Carter's total cost? Explain why your
estimate is reasonable.

• Dividing by a Decimal Number

Power Up

facts   Power Up K

mental math

   a. Number Sense: 2×250

   b. Number Sense: 4×125

   c. Estimation: The textbook is $11\frac{1}{8}$ in. long and $8\frac{1}{8}$ in. wide. Round each length to the nearest inch, and then use your estimates to find the approximate perimeter of the book cover.

   d. Geometry: What is the area of a patio that is 15 ft long and 10 ft wide?

   e. Percent: What number is 10% of 20?

   f. Percent: What number is 10% more than 20?

   g. Percent: What number is 10% less than 20?

   h. Roman Numerals: Compare: MCMXCIX $\bigcirc$ MM

problem solving

Choose an appropriate problem-solving strategy to solve this problem. Makayla erased some of the digits from a multiplication problem and gave it to Connor as a problem-solving exercise. Copy Makayla's multiplication problem and find the missing digits for Connor.

$$\begin{array}{r} \_7 \\ \times \ \_\_ \\ \hline 5\_6 \end{array}$$

New Concept

We have practiced dividing decimal numbers by whole numbers. In this lesson we will practice dividing decimal numbers by decimal numbers.

Thinking Skill

Generalize

How is dividing a decimal by a whole number the same as dividing a decimal by a decimal? How is it different?

The two problems below are different in an important way.

$$3\overline{)0.12} \qquad 0.3\overline{)0.12}$$

The problem on the left is division **by a whole number.** The problem on the right is division **by a decimal number.**

When dividing by a decimal number with pencil and paper, we take an extra step. Before dividing, we shift the decimal points so that we are dividing by a whole number instead of by a decimal number.

$$0.3\overline{)0.12}$$

We move the decimal point of the divisor so that it becomes a whole number. Then we move the decimal point of the dividend the same number of places. The decimal point in the quotient will be straight up from the new location of the dividend's decimal point. To remember how to divide by a decimal number, we may think, "Over, over, and up."

$$\overset{\text{up}}{0.3\overline{)0.12}}$$
over over

To help us understand why this procedure works, we will write "0.12 divided by 0.3" with a division bar.

$$\frac{0.12}{0.3}$$

Notice that we can change the divisor, 0.3, into a whole number by multiplying by 10. So we multiply by $\frac{10}{10}$ to make an equivalent division problem.

$$\frac{0.12}{0.3} \times \mathbf{\frac{10}{10}} = \frac{1.2}{3}$$

Multiplying by $\frac{10}{10}$ moves both decimal points "over." Now the divisor is a whole number and we can divide.

$$3\overline{)1.2} \quad \overset{0.4}{}$$

We will add this memory cue to the decimals chart. In the last column, "÷ by decimal (*D*)" means "division by a decimal number."

Decimals Chart

Operation	+ or −	×	÷ by whole (W)	÷ by whole (D)
Memory cue	line up . $\pm$ . .	×; then count . _ $\times$. _ . _ _	up $W\overline{)\,.\,}^{\,.}$	over, over, up $D\overline{)\,\smile\,}^{\,.}$
You may need to:				
• Place a decimal point on the end of whole numbers.				
• Fill each empty place with a zero.				

Example

Divide: $0.6\overline{)2.34}$

We are dividing by the decimal number 0.6. We change 0.6 into a whole number by moving its decimal point "over." We also move the decimal point in the dividend "over." The decimal point in the quotient will be "straight up" from the new location of the decimal point in the division box.

$$
\begin{array}{r}
3.9 \\
0.6\overline{)2.3\,4} \\
1\,8 \\
\hline
5\,4 \\
5\,4 \\
\hline
0
\end{array}
$$

Verify Demonstrate how to check the answer.

Lesson Practice

Divide:

a. $0.3\overline{)1.2}$ **b.** $0.3\overline{)0.42}$ **c.** $1.2\overline{)0.24}$

d. $0.4\overline{)0.24}$ **e.** $0.4\overline{)5.6}$ **f.** $1.2\overline{)3.6}$

g. $0.6\overline{)2.4}$ **h.** $0.5\overline{)0.125}$ **i.** $1.2\overline{)2.28}$

Written Practice

Distributed and Integrated

*1. Copy the decimals chart in this lesson.
(119)

*2. The ages of five neighborhood friends are 9, 8, 7, 6, and 5 years. What
(50) is the average age of the friends?

3. At the wildlife park there were lions, tigers, and bears. There were
(49) 24 bears. If there were twice as many lions as tigers and twice as many
tigers as bears, how many lions were there?

4. Joey has $18.35. Raimi has $22.65. They want to put their money
(49) together to buy a car that costs $16,040. How much more money
do they need?

*5. **a.** **Multiple Choice** Which numbers below do *not* show a line of
(105) symmetry?

A $\;8\;$ B $\;3\;$ C $\;5\;$ D $\;2\;$
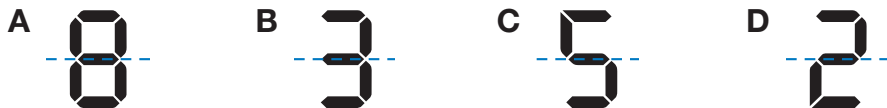
b. Which figure **A–D** does *not* have rotational symmetry?

*6. Write the mixed number $3\frac{1}{3}$ as an improper fraction. Then multiply the
(91, 113) improper fraction by $\frac{3}{4}$. Remember to simplify your answer.

7. (Conclude) Refer to quadrilateral *ABCD* to answer parts
(31, 45) **a** and **b.**
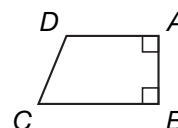
a. Which angle appears to be an obtuse angle?

b. What type of quadrilateral is quadrilateral *ABCD?*

*8. $3\frac{1}{2}$ ***9.** $2\frac{1}{6}$ ***10.** $5\frac{5}{6}$ ***11.** $4\frac{2}{3}$
(116)   (116)   (116)   (116)

$+\,1\frac{1}{3}$ $+\,1\frac{1}{2}$ $-\,1\frac{1}{2}$ $-\,1\frac{1}{4}$

*12. $6\overline{)0.0144}$ ***13.** $5\overline{)1.2}$ **14.** $12\overline{)1800}$
(117)   (118)   (34, 92)

*15. $0.3\overline{)0.24}$ ***16.** $50\overline{)1000}$ **17.** $1.2\overline{)0.180}$
(119)   (34, 54)   (119)

*18. Divide mentally:
(118)
  a. $0.5 \div 10$ **b.** $0.5 \div 100$

19. $(3 - 1.6) - 0.16$
(24, 102)

20. 0.12 ***21.** 0.12 **22.** 75
(110) $\times\,0.30$ (111) $\times\quad 10$ (51) $\times\,48$

*23. $4 \times \frac{3}{8}$ ***24.** $4 \div \frac{3}{8}$
(89, 91)   (96)

*25. a. What is the perimeter of this rectangle?
(53, 72)

  b. What is the area of this rectangle?

$\frac{1}{2}$ ft

$\frac{1}{3}$ ft
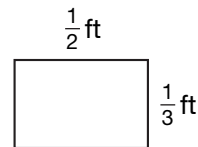
26. What is the volume of a room that is 10 feet wide, 12 feet long, and
(103) 8 feet high?

*27. Two squares are joined to form this hexagon. Refer to the
(53, 115) figure to answer parts **a** and **b**.

10 ft

5 ft

  a. What is the area of the hexagon?

  b. Copy the hexagon and show the lengths of the six sides. Then find
  the perimeter of the hexagon.

*28. Trevor polled fifth graders to find out how many items they put in their
(Inv. 5) backpacks. The data below shows the results of his poll.

3, 2, 5, 5, 8, 4, 3, 4, 7, 2, 4, 8, 5, 10, 5

  a. Display the data in a line plot.

  b. Find the median of the data.

  c. Find the mode or modes of the data.

  d. Find the range of the data.

29. The school bus Nico rides is designed to carry 48 students. Thirteen
(49) students were already on the bus when Nico boarded the bus this
morning. Eleven more students boarded the bus while Nico was riding
to school. How many seats on the bus were empty when it arrived at
school?

30. The number of students enrolled at five different elementary schools is
(33, 49) shown below.

341   307   462   289   420

  a. **Estimate** Estimate the number of students altogether who
  attend the schools.

  b. Use your estimated total to find the approximate average number
  of students in each school.

• Multiplying Mixed Numbers

Power Up

facts

Power Up K

mental math

a. **Measurement:** The mass of a softball is about 200 grams. What is the approximate mass of 5 softballs?

b. **Measurement:** Brianna poured out 375 mL from the 1-liter bottle of water. How many mL were left in the bottle?

c. **Percent:** How much is 25% of $60?

d. **Percent:** How much is 25% less than $60?

e. **Percent:** How much is 25% more than $60?

f. **Number Sense:** A score is a set of 20. Two score is 40. Three score equals how many dozen?

g. **Calculation:** 5^2, $- 5$, $\times 3$, $+ 3$, $\div 9$, $\times 4$, $- 1$, $\div 3$

h. **Roman Numerals:** Write the current year in Roman numerals.

problem solving

Choose an appropriate problem-solving strategy to solve this problem. The word BOB has a horizontal line of symmetry because each of its letters has a horizontal line of symmetry.

BOB

Write "BOB" on a sheet of paper. Fold the upper half of the word along the line of symmetry. The lower half of the word should look like this: ꓭOꓭ

Place the paper against a reflective surface or mirror. Notice that the upper half of the word "reappears."

Other words that have a horizontal line of symmetry are BED, BOOK, HE and HI. Try the activity again using one of these words. Explain how this "trick" works.

Thinking Skill

Connect

What are the steps for multiplying fractions?

To multiply mixed numbers, we change the mixed numbers to improper fractions before we multiply.

$$2\frac{1}{2} \times 1\frac{2}{3}$$

First change the mixed numbers to improper fractions.

$$\frac{5}{2} \times \frac{5}{3} = \frac{25}{6} \qquad \frac{25}{6} = 4\frac{1}{6}$$

Then multiply.   Then simplify.

Example 1

Multiply: $\frac{1}{5} \times 4\frac{1}{2}$

First we write the mixed number as an improper fraction. When both numbers are written as fractions, we multiply. We find that $\frac{1}{5}$ of $4\frac{1}{2}$ is $\frac{9}{10}$.

Justify Can we use $\frac{9}{10} \div \frac{9}{2}$ to check the answer? Why or why not?

$$\frac{1}{5} \times 4\frac{1}{2}$$

$$\downarrow$$

$$\frac{1}{5} \times \frac{9}{2} = \frac{9}{10}$$

Example 2

Multiply: $3 \times 2\frac{1}{3}$

We write both numbers as improper fractions; then we multiply.

$$3 \times 2\frac{1}{2}$$

$$\frac{3}{1} \times \frac{7}{3} = \frac{21}{3} = 7$$

We simplified the result to find that the product is **7.** We found our answer by multiplying. We find the same answer if we add:

$$2\frac{1}{3} + 2\frac{1}{3} + 2\frac{1}{3} = 6\frac{3}{3} = 7$$

Lesson Practice   Multiply:

a. $1\frac{1}{2} \times 1\frac{3}{4}$

b. $3\frac{1}{2} \times 1\frac{2}{3}$

c. $3 \times 2\frac{1}{2}$

d. $4 \times 3\frac{2}{3}$

e. $\frac{1}{3} \times 2\frac{1}{3}$

f. $\frac{1}{6} \times 2\frac{5}{6}$

*1. Copy the decimals chart from Lesson 119.
(119)

2. **a.** Name this figure.
(83)

   b. How many faces does this figure have?

   c. How many vertices does this figure have?

   d. Which faces are congruent and parallel?
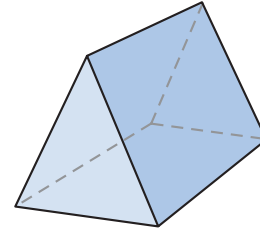
3. **Represent** Write the following sentence using digits and symbols:
(4, 15)
   The sum of two and two equals the product of two and two.

*4. **Multiple Choice** Which of these is *not* equal to $\frac{1}{2}$?
(71, 100)
   A 0.5 **B** 50% **C** 0.50 **D** 0.05

5. **Explain** Lily cares for a cat and a kitten. The cat weighs
(101) $7\frac{3}{4}$ pounds. What is a reasonable estimate of the kitten's weight if the
   cat and the kitten together weigh about 11 pounds? Explain why your
   estimate is reasonable.

6. Jillian can type 4 pages in 1 hour. At that rate, how long will it take her
(49) to type 100 pages?

7. In rectangle *ABCD,* $\overline{BC}$ is twice the length of $\overline{AB}$. Segment
(53, 72) *AB* is 3 inches long.
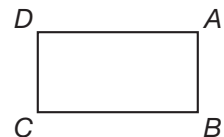
   a. What is the perimeter of the rectangle?

   b. What is the area of the rectangle?

   c. Name two pairs of parallel sides.

   d. Name two pairs of perpendicular sides.

*8. Emilio is about to roll a standard number cube.
(57, 80)

     a. What is the probability that he will get a prime number in one roll?

     b. What is the chance that he will not get a prime number in one roll?

9. A decagon has how many more sides than a pentagon?
(32)

10. What is the average of 2, 4, 6, and 8?
(50)

11. *QR* equals *RS. ST* is 5 cm. *RT* is 7 cm. Find *QT.*
(61)

$$Q \qquad R \qquad S \qquad\qquad\qquad\qquad T$$
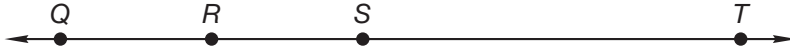
12. $38.248 + 7.5 + 37.23 + 15$
(99)

13. $\$6 - (\$1.49 - 75¢)$ **14.** 2.4×100
(24, 70)   (111)

15. 0.24×0.12 **16.** 25×50
(110)   (51)

*17. $8\overline{)0.1000}$ ***18.** $0.5\overline{)4.35}$ ***19.** $12\overline{)1440}$
(117)   (119)   (92,
                                                                               132)

*20. $3\frac{1}{3}$ ***21.** $\frac{3}{7}$ ***22.** $6\frac{14}{15}$ ***23.** $\frac{4}{5}$
(116)   (116)   (116)   (116)

      $+\ 7\frac{3}{4}$ $+\ \frac{1}{2}$ $-\ 1\frac{1}{5}$ $-\ \frac{1}{3}$

*24. $\frac{1}{2} \times 3\frac{1}{3}$ ***25.** $4 \times 2\frac{1}{2}$
(120)   (120)

26. a. What is the area of a bedroom that is 3 meters wide and
(109)   4.5 meters long?

      b. What is the perimeter?

4.5 m

3 m
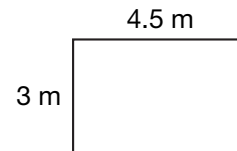
*27. What is the volume of a drawer that is 2 ft by 1.5 ft by 0.5 ft?
(103,
109)

0.5 ft

1.5 ft

2 ft
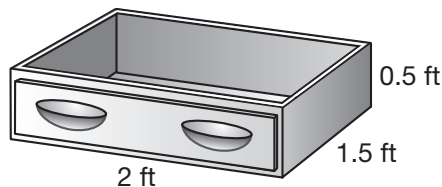
*28. Refer to the figure at right to solve parts **a–c.**
(Inv. 4)
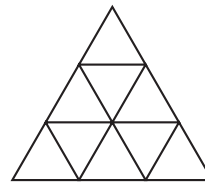
    a. (**Analyze**) The perimeter of each small equilateral triangle is 6 inches. What is the perimeter of the large equilateral triangle?

    b. The area of one small triangle is what percent of the area of the large triangle?

    c. (**Conclude**) A sequence of triangle patterns is shown below. Draw the next triangle in the pattern on your paper. How many small triangles form the large triangle in your drawing?

$$\triangle, \quad \triangle\!\!\triangle, \quad \triangle\!\triangle\!\triangle, \quad \underline{\hspace{2cm}}, \ldots$$
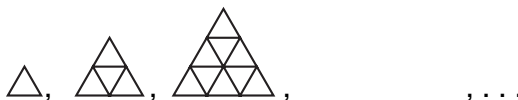
29. Four-hour admission to an outdoor water park costs $12.50 per person.
(49) Gary and three friends plan to visit the park. They have a discount coupon for $2 off each person's admission. What is the total cost of the tickets?

30. ✎ (**Justify**) Wyatt estimated the quotient of 189 ÷ 5 to be about 40.
(62) Did Wyatt make a reasonable estimate? Explain why or why not.

Early Finishers
Real-World Connection

Ms. Valdez's car is being repaired. To get to and from school, her daughter Paula will walk a total of $\frac{7}{8}$ of a mile each day for 9 days.

    a. How far will Paula walk altogether? Estimate and then find the actual product.

    b. Is your answer reasonable?

    c. If Paula walked to school for 3 full school weeks, how far would she walk in all?

Focus on
• Tessellations

Archaeologists know that people have been using tiles to make mosaics and to decorate homes and other buildings since about 4000 B.C. The Romans called these tiles *tesselae,* from which we get the word **tessellation** (tiling). A tessellation is the repeated use of shapes to fill a flat surface without gaps or overlaps. Below are some examples of tessellations. We say that the polygons in these figures *tessellate;* in other words, they tile a plane.
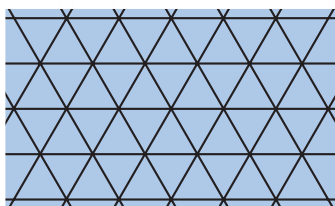
Figure 1   Figure 2
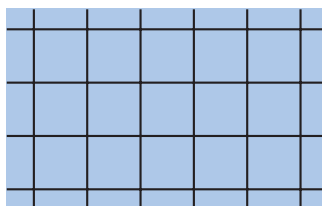
These tessellations are called *regular tessellations* because one regular polygon is used again and again to tile the plane. Although the same shape is used repeatedly in regular tessellations, the orientation of the shape may vary from tile to tile. In Figure 1, for example, we see that all the triangles are congruent, but that alternate triangles are rotated 180° (half of a turn).

Now look at a vertex in each figure and count the number of polygons that meet at the vertex. Notice that a certain number of polygons meet at each vertex in each tessellation.

    1. How many triangles meet at each vertex in Figure 1?

    2. How many squares meet at each vertex in Figure 2?

Only a few regular polygons tessellate. Here is an example of a regular polygon that does not tessellate:
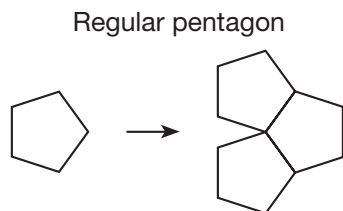
Regular pentagon

We see that the regular pentagon on the left will not fit into the gap formed by the other pentagons. Therefore, a regular pentagon does not tessellate.

3. Multiple Choice Which of these regular polygons tessellates? Draw a tessellation that uses that polygon.

A   B   C   D
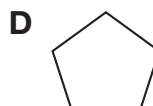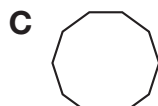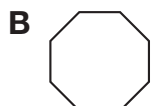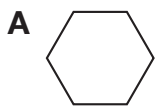
There are some combinations of regular polygons that tessellate. Below is an example of a tessellation that combines regular hexagons and equilateral triangles. A tiling composed of two or more regular polygons such as this is called a *semiregular tessellation.*
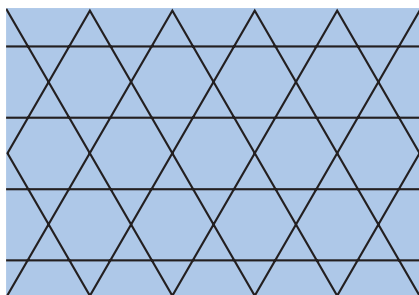
4. Multiple Choice Which two of these regular polygons could combine to tile a plane? Draw a picture that shows the tessellation.

A   B   C   D
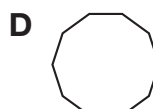
Many polygons that are not regular polygons can tile a plane. In fact, every triangle can tile a plane, and every quadrilateral can tile a plane. Here is an example using each type of polygon:
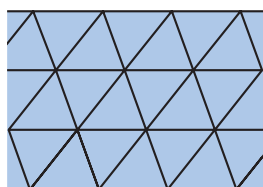
Triangle   Quadrilateral
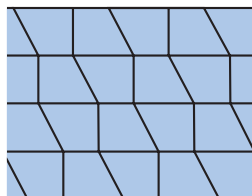
Notice in both examples that the tiles are congruent, but that alternate tiles are rotated 180°.

Activity 1

Triangle and Quadrilateral Tessellations

Materials needed:
- **Lesson Activity 45**
- scissors

5. Carefully cut out the triangles on **Lesson Activity 45.** On your desk, arrange the triangles like tiles so that the vertices of six triangles meet at a point and the sides align without gaps or overlapping. Do not flip (reflect) the triangles to make them fit.

6. Carefully cut out the quadrilaterals on **Lesson Activity 45.** When tiling with quadrilaterals, arrange the quadrilaterals so that the vertices of four quadrilaterals meet at a point.

Some polygons that tessellate can be carefully altered and fitted together to form intricate tessellations. In the example below, we start with an equilateral triangle and alter one side by cutting out a piece of the triangle. Then we attach the cutout piece to another side of the triangle. If we make several congruent figures, we can fit them together to tile a surface.

Equilateral triangle   First side altered

Second side altered

Tessellation
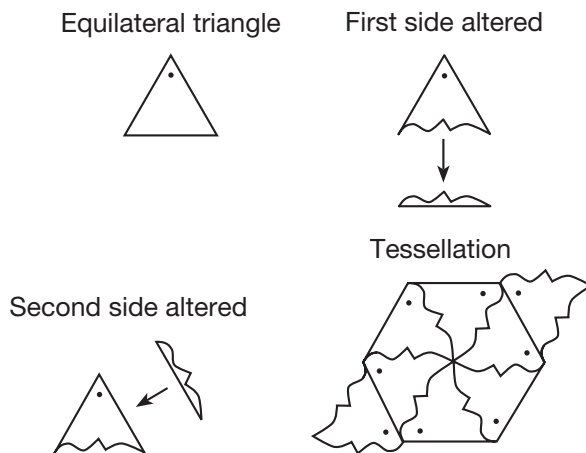
In the next example, we start with a square. We alter one side of the square and then make the corresponding alteration to the opposite side. Then we alter a third side of the square and make the corresponding alteration to the remaining side. Congruent copies of the figure tessellate.

Square   First side altered   Opposite side altered

Third side altered   Opposite side altered   Tessellation
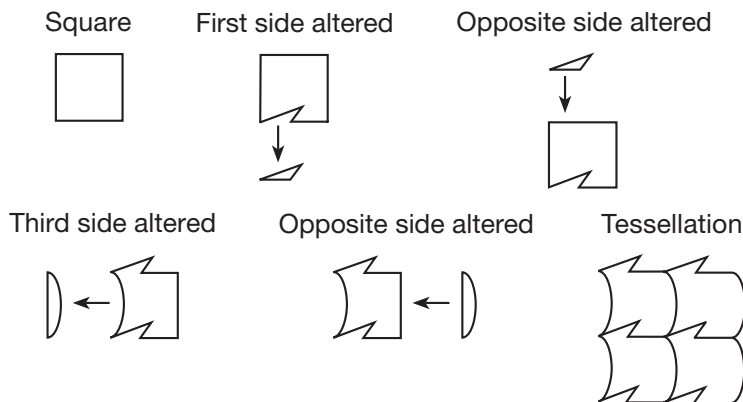
Activity 2

Creating Tessellations with Altered Figures

Materials needed:
- **Lesson Activity 46**
- ruler
- several sheets of unlined paper
- scissors
- glue or tape
- colored pencils or crayons (optional)

In this activity you will alter a triangle or a square and then use the resulting figure to create a tessellation. First choose one of the two shapes at the bottom of **Lesson Activity 46.** Trace that figure onto a blank sheet of paper, using a ruler to keep the sides of the traced figure straight. Then cut out the traced figure with scissors. Now follow the set of directions below that applies to the shape you chose.

Triangle

Step 1: Alter one side of the triangle by cutting a section from the shape. Be sure to cut out only one section. (Do not cut several pieces from the shape.)

Step 2: Tape the cutout section to another side of the figure. Use scissors to cut away excess tape.

Step 3: Trace the altered figure 8 to 12 times onto blank paper. You may color the figures you traced with colored pencils or crayons.

Step 4: Use scissors to cut out the traced figures.

Step 5: Fit the figures together to tile a portion of the box provided on **Lesson Activity 46.**

Step 6: Glue or tape the tiles into place.

Square

Step 1: Alter one side of the square by cutting a section from the shape. Be sure to cut out only one section. (Do not cut several pieces from the shape.)

Step 2: Tape the cutout section to the opposite side of the figure. Use scissors to cut away excess tape.

Step 3: Optional: Repeat Steps 1 and 2 to alter the remaining two sides of the figure.

Step 4: Trace the altered figure 8 to 12 times onto blank paper. You may color the figures you traced with colored pencils or crayons.

Step 5: Use scissors to cut out the traced figures.

Step 6: Fit the figures together to tile a portion of the box provided on **Lesson Activity 46.**

Step 7: Glue or tape the tiles into place.

Investigate Further

a. Find examples of tessellations in floor tiles at school or at home. Trace or copy the patterns, and bring them to class to display.

b. Search the Internet for information about tessellations. Share pictures and/or information you found with the rest of the class.

c. These figures have been sorted into a group by one common characteristic.
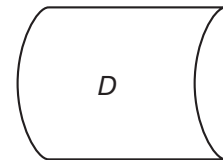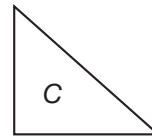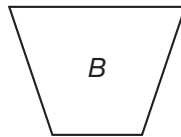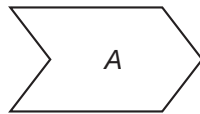
Figures *E* and *F* do not belong in the group above.
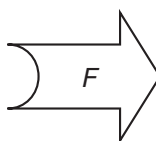
Draw a figure that belongs in the first group. Then explain how you found your answer and why your answer is reasonable.

• Roman Numerals Through 39

New Concept

Roman numerals were used by the ancient Romans to write numbers. Today Roman numerals are still used to number such things as book chapters, movie sequels, and Super Bowl games. We might also find Roman numerals on clocks and buildings.

Some Roman numerals are

> I   which stands for 1
>
> V   which stands for 5
>
> X   which stands for 10

The Roman numeral system does not use place value. Instead, the values of the numerals are added or subtracted, depending on their position. For example,

> II   means   1 plus 1, which is 2.   (II does not mean "11.")

Below we list the Roman numerals for the numbers 1 through 20. Study the patterns.

1 = I	11 = XI
2 = II	12 = XII
3 = III	13 = XIII
4 = IV	14 = XIV
5 = V	15 = XV
6 = VI	16 = XVI
7 = VII	17 = XVII
8 = VIII	18 = XVIII
9 = IX	19 = XIX
10 = X	20 = XX

The multiples of 5 are 5, 10, 15, 20, ... The numbers that are one less than these (4, 9, 14, 19, . . .) have Roman numerals that involve subtraction.

$$4 = \text{IV} \quad \text{(“one less than five”)}$$
$$9 = \text{IX} \quad \text{(“one less than ten”)}$$
$$14 = \text{XIV} \quad \text{(ten plus “one less than five”)}$$
$$19 = \text{XIX} \quad \text{(ten plus “one less than ten”)}$$

In each case where a smaller Roman numeral (I) precedes a larger Roman numeral (V or X), we subtract the smaller number from the larger number.

Example

a. **Write XXVII in our number system.[1]**

b. **Write 34 in Roman numerals.**

a. We can break up the Roman numeral and see that it equals 2 tens plus 1 five plus 2 ones.

$$\text{XX} \quad \text{V} \quad \text{II}$$
$$20 + 5 + 2 = \mathbf{27}$$

b. We think of 34 as “30 plus 4.”

$$30 + 4$$
$$\text{XXX} \quad \text{IV}$$

So the Roman numeral for 34 is **XXXIV.**

Lesson Practice   Write the Roman numerals for 1 to 39 in order.

• Roman Numerals Through Thousands

New Concept

We have practiced using these Roman numerals:

I	V	X

With these numerals we can write counting numbers up to XXXIX (39). To write larger numbers, we must use the Roman numerals L (50), C (100), D (500), and M (1000). The table below shows the different Roman numeral "digits" we have learned, as well as their respective values.

Numeral	I	V	X	L	C	D	M
Value	1	5	10	50	100	500	1000

Example

Write each Roman numeral in our number system:

a. LXX   **b. DCCL**   **c. XLIV**   **d. MMI**

a. LXX is 50 + 10 + 10, which is **70.**

b. DCCL is 500 + 100 + 100 + 50, which is **750.**

c. XLIV is "10 less than 50" plus "1 less than 5," that is, 40 + 4 = **44.**

d. MMI is 1000 + 1000 + 1, which is **2001.**

Lesson Practice

Write each Roman numeral in our number system:

a. CCCLXII   **b.** CCLXXXV   **c.** CD

d. XLVII   **e.** MMMCCLVI   **f.** MCMXCIX

A

acute angle
(31)

An angle whose measure is more than 0° and less than 90°.

acute angle

right angle   obtuse angle
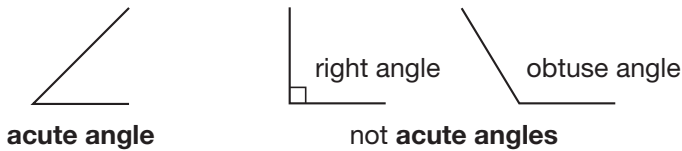
not **acute angles**

*An **acute angle** is smaller than both a right angle and an obtuse angle.*

ángulo agudo

Ángulo que mide más de 0° y menos de 90°.

*Un **ángulo agudo** es menor que un ángulo recto y que un ángulo obtuso.*

acute triangle
(36)

A triangle whose largest angle measures less than 90°.

right triangle   obtuse triangle

acute triangle

not **acute triangles**
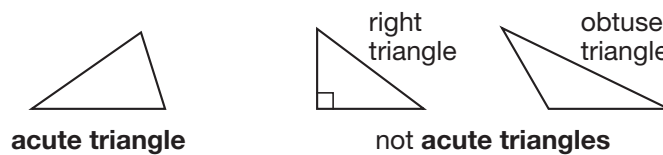
triángulo acutángulo

Triángulo cuyo ángulo mayor es menor que 90°.

addend
(6)

Any one of the numbers added in an addition problem.

$7 + 3 = 10$ The **addends** in this problem are 7 and 3.

sumando

Uno de dos o más números que se suman en un problema de suma.

*Los **sumandos** en este problema son el 7 y el 3.*

adjacent sides
(45)

Sides that intersect.

adjacent sides
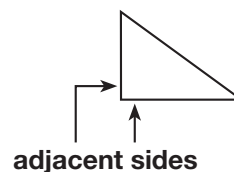
lados adyacentes

Lados que intersecan.

algorithm
(6)

Any process for solving a mathematical problem.

*In the addition **algorithm** we add the ones first, then the tens, and then the hundreds.*

algoritmo

Cualquier proceso para resolver un problema matemático.

*En el **algoritmo** de la suma, primero sumamos las unidades, después las decenas y al final las centenas.*

a.m.
(29)

The period of time from midnight to just before noon.

*I get up at 7 **a.m.**, which is 7 o'clock in the morning.*

a.m.

El período de tiempo desde la medianoche hasta antes del mediodía.

*Me levanto a las 7 **a.m.**, que son las 7 en punto de la mañana.*

angle (31)	The opening that is formed when two lines, line segments, or rays intersect. *These line segments form an **angle.***
ángulo	Abertura que se forma cuando se intersecan dos rectas, rayos o segmentos de recta. *Estos segmentos de recta forman un **ángulo.***
area (72)	The number of square units needed to cover a surface. 5 in. 2 in. *The **area** of this rectangle is 10 square inches.*
área	El número de unidades cuadradas que se necesitan para cubrir una superficie. *El **área** de este rectángulo mide 10 pulgadas cuadradas.*
arithmetic sequence (Inv. 4)	A sequence in which each term is found by adding a fixed amount to the previous term. +3 +3 +3 +3 3, 6, 9, 12, 15, … *This **arithmetic sequence** counts up by 3s.*
secuencia aritmética	Una secuencia en la que cada término se encuentra sumando una cantidad fija al término anterior. *Esta **secuencia aritmética** cuenta de tres en tres.*
array (13, 80)	A rectangular arrangement of numbers or symbols in columns and rows. X X X X X X *This is a 3-by-4 **array** of Xs.* X X X *It has 3 columns and 4 rows.* X X X
matriz	Un arreglo rectangular de números o símbolos en columnas y filas. *Esta es una **matriz** de X de 3-por-4. Tiene 3 columnas y 4 filas.*
Associative Property of Addition (24)	The grouping of addends does not affect their sum. In symbolic form, $a + (b + c) = (a + b) + c$. Unlike addition, subtraction is not associative. $(8 + 4) + 2 = 8 + (4 + 2)$ $(8 - 4) - 2 \neq 8 - (4 - 2)$ *Addition is **associative.*** *Subtraction is not **associative.***
propiedad asociativa de la suma	La agrupación de los sumandos no altera la suma. En forma simbólica, $a + (b + c) = (a + b) + c$. A diferencia de la suma, la resta no es asociativa. *La suma es **asociativa.*** *La resta no es **asociativa.***
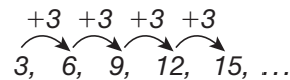
Associative Property of Multiplication (24)	The grouping of factors does not affect their product. In symbolic form, $a \times (b \times c) = (a \times b) \times c.$ Unlike multiplication, division is not associative.
	$(8 \times 4) \times 2 = 8 \times (4 \times 2)$ $(8 \div 4) \div 2 \neq 8 \div (4 \div 2)$
	*Multiplication is **associative.*** *Division is not **associative.***
propiedad asociativa de la multiplicación	La agrupación de los factores no altera el producto. En forma simbólica, $a \times (b \times c) = (a \times b) \times c.$ A diferencia de la multiplicación, la división no es asociativa.
	*La multiplicación es **asociativa.*** *La división no es **asociativa.***

average (50)	The number found when the sum of two or more numbers is divided by the number of addends in the sum; also called *mean.*
	*To find the **average** of the numbers 5, 6, and 10, first add.*
	$5 + 6 + 10 = 21$
	Then, since there were three addends, divide the sum by 3.
	$21 \div 3 = 7$
	*The **average** of 5, 6, and 10 is 7.*
promedio	Número que se obtiene al dividir la suma de un conjunto de números entre la cantidad de sumandos; también se le llama *media.*
	*Para calcular el **promedio** de los números 5, 6 y 10, primero se suman.*
	Como hay tres sumandos, se divide la suma entre 3.
	*El **promedio** de 5, 6 y 10 es 7.*

B

bar graph (Inv. 5, Inv. 7)	A graph that uses rectangles (bars) to show numbers or measurements.
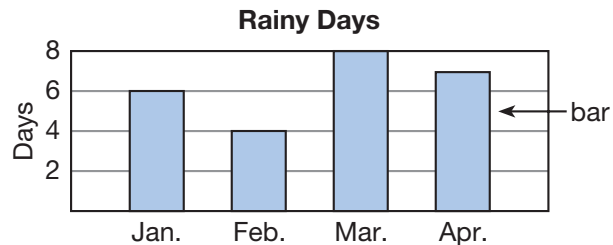
Rainy Days

	This **bar graph** shows how many rainy days there were in each of these four months.
gráfica de barras	Una gráfica que usa rectángulos (barras) para mostrar números o medidas.
	*Esta **gráfica de barras** muestra cuantos días lluviosos hubo en cada uno de estos cuatro meses.*

base *(78, 83)*	**1.** The lower number in an exponential expression. $$base \longrightarrow 5^3 \longleftarrow exponent$$ 5^3 *means* $5 \times 5 \times 5$, *and its value is 125.* **2.** A designated side or face of a geometric figure. base base base
base	**1.** El número inferior en una expresión exponencial. **2.** Un lado o cara designado de una figura geométrica.
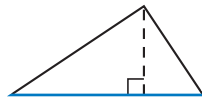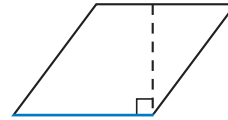
base-ten system *(7)*	A place-value system in which each place value is 10 times larger than the place value to its right. *The decimal system is a **base-ten system.***
sistema base diez	Un sistema de valor posicional en el que el valor de la posición es 10 veces mayor que el valor de la posición a su derecha. *El sistema decimal es un **sistema de base diez.***

C

capacity *(85)*	The amount of liquid a container can hold. *Cups, gallons, and liters are units of **capacity.***
capacidad	Cantidad de líquido que puede contener un recipiente. *Tazas, galones y litros son medidas de **capacidad.***

cardinal number(s) *(7)*	The counting numbers 1, 2, 3, 4, …
número(s) cardinal(es)	Los números de conteo 1, 2, 3, 4, ...

Celsius *(27)*	A scale used on some thermometers to measure temperature. *On the **Celsius** scale, water freezes at 0°C and boils at 100°C.*
Celsius	Escala que se usa en algunos termómetros para medir la temperatura. *En la escala **Celsius,** el agua se congela a 0°C y hierve a 100°C.*

center *(53)*	The point inside a circle from which all points on the circle are equally distant. 2 in. *The **center** of circle A is 2 inches* A *from every point on the circle.*
centro	Punto interior de un círculo o esfera, que equidista de cualquier punto del círculo o de la esfera. *El **centro** del círculo A está a 2 pulgadas de cualquier punto del círculo.*
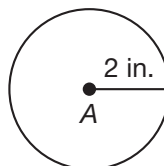
centigrade (27)	A metric system temperature scale with one hundred gradations, or degrees, between the freezing and boiling points of water. *The Celsius scale is a **centigrade** scale.*
centígrado	Una escala de temperatura del sistema métrico con cien gradaciones o grados, entre el *punto de ebullición y el de congelación del agua.* *La escala de Celsius es una escala de **centígrados.***
centimeter (44, 65)	One hundredth of a meter. *The width of your little finger is about one **centimeter.***
centímetro	Una centésima de un metro. *El ancho de tu dedo meñique mide aproximadamente un **centímetro.***
century (28)	A period of one hundred years. *The years 2001–2100 make up one **century.***
siglo	Un período de tiempo de cien años. *Los años del 2001 al 2100 forman un **siglo.***
certain (57)	We say that an event is **certain** when the event's probability is 1. This means the event will definitely occur.
seguro	Decimos que un evento es *seguro* cuando la probabilidad de que el evento ocurra es 1. Esto significa que el evento definitivamente va a ocurrir.
chance (57)	A way of expressing the likelihood of an event; the probability of an event expressed as a percentage. *The **chance** of rain is 20%. It is not likely to rain.* *There is a 90% **chance** of snow. It is likely to snow.*
posibilidad	Modo de expresar la probabilidad de ocurrencia de un suceso; la probabilidad de un suceso expresada como porcentaje. *La **posibilidad** de lluvia es del 20%. Es poco probable que llueva.* *Hay un 90% de **posibilidad** de nieve. Es muy probable que nieve.*
circle (53)	A closed, curved shape in which all points on the shape are the same distance from its center.
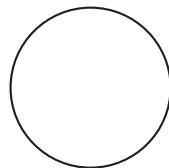
circle

círculo	Una figura cerrada y curva en la cual todos los puntos están a la misma distancia de su centro.

circle graph *(Inv. 7)*	A graph made of a circle divided into sectors. Also called *pie chart* or *pie graph*.

Shoe Colors of Students
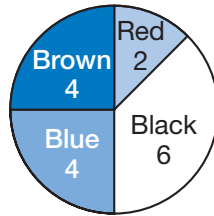
*This **circle graph** displays data on students' shoe color.*

gráfica circular	Una gráfica circular está formada por un círculo dividido en sectores. También llamada *diagrama circular*. *Esta **gráfica circular** representa los datos del color de zapatos de los estudiantes.*

---

circumference *(53)*	The distance around a circle; the perimeter of a circle.
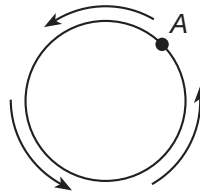
*If the distance from point A around to point A is 3 inches, then the **circumference** of the circle is 3 inches.*

circunferencia	La distancia alrededor de un círculo. Perímetro de un círculo. *Si la distancia desde el punto A alrededor del círculo hasta el punto A es 3 pulgadas, entonces la **circunferencia** del círculo mide 3 pulgadas.*

---

cluster *(Inv. 5)*	A group of data points that are very close together.
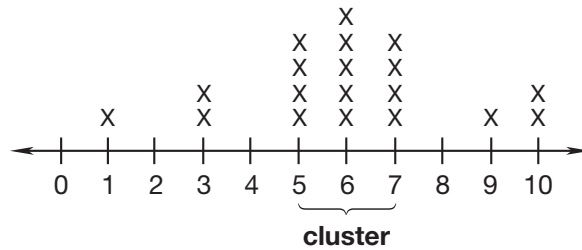
cúmulo	Un grupo de puntos de datos que están muy cerca uno del otro.

---

common denominators *(41)*	Denominators that are the same. *The fractions $\frac{2}{5}$ and $\frac{3}{5}$ have **common denominators.***
denominadores comunes	Denominadores que son iguales. *Las fracciones $\frac{2}{5}$ y $\frac{3}{5}$ tienen **denominadores comunes.***

---

common fraction *(67)*	A fraction with whole-number terms.

$$\frac{1}{2} \quad \frac{5}{7} \quad \frac{3}{4} \qquad\qquad \frac{1.2}{2.4} \quad \frac{3}{4.5} \quad \frac{2.5}{3}$$

common fractions   not ***common fractions***

fracción común	Una fracción con términos que son números enteros.

common year *(28)*	A year with 365 days; not a leap year.
	*The year 2000 is a leap year, but 2001 is a **common year.*** *In a **common year** February has 28 days. In a leap year it has 29 days.*
año común	Un año con 365 días; no un año bisiesto.
	*El año 2000 es un año bisiesto, pero el año 2001 es un **año común.***
	*En un **año común** febrero tiene 28 días. En un año bisiesto tiene 29 días.*

Commutative Property of Addition *(6)*	Changing the order of addends does not change their sum. In symbolic form, $a + b = b + a$. Unlike addition, subtraction is not commutative.
	$8 + 2 = 2 + 8$ $8 - 2 \neq 2 - 8$
	*Addition is **commutative.*** *Subtraction is not **commutative.***
propiedad conmutativa de la suma	El orden de los sumandos no altera la suma. En forma simbólica, $a + b = b + a$. A diferencia de la suma, la resta no es conmutativa.
	*La suma es **conmutativa.*** *La resta no es **conmutativa.***

Commutative Property of Multiplication *(15)*	Changing the order of factors does not change their product. In symbolic form, $a \times b = b \times a$. Unlike multiplication, division is not commutative.
	$8 \times 2 = 2 \times 8$ $8 \div 2 \neq 2 \div 8$
	*Multiplication is **commutative.*** *Division is not **commutative.***
propiedad conmutativa de la multiplicación	El orden de los factores no altera el producto. En forma simbólica, $a \times b = b \times a$. A diferencia de la multiplicación, la división no es conmutativa.
	*La multiplicación es **conmutativa.*** *La división no es **conmutativa.***

comparative graph *(93)*	A method of displaying data, usually used to compare two or more related sets of data.

Department Store Sales
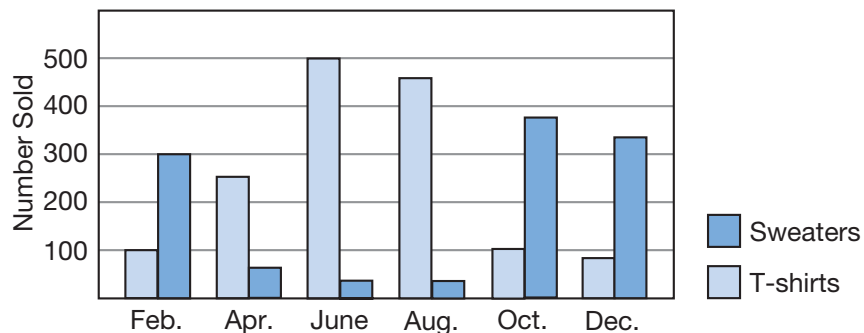
*This **comparative graph** compares how many sweaters were sold with how many T-shirts were sold in each of these six months.*

gráfica comparativa	Un método para mostrar datos, usualmente usado para comparar dos o más conjuntos de datos relacionados.
	*Esta **gráfica comparativa** compara cuantos suéteres se vendieron con cuántas camisetas se vendieron en cada uno de estos seis meses.*

comparison symbol *(4)*	A mathematical symbol used to compare numbers. ***Comparison symbols** include the equal sign (=) and the "greater than/less than" symbols (> or <).*
signo de comparación	Un símbolo matemático que se usa para comparar números. *Los **signos de comparación** incluyen el signo de igual (=) y los signos "mayor que/menor que" (> ó <).*
compatible numbers *(33)*	Numbers that are close in value to the actual numbers and are easy to add, subtract, multiply, or divide mentally.
números compatibles	Números que están cerca en valor a los números originales y que son fáciles de sumar, restar, multiplicar o dividir mentalmente.
composite number *(80)*	A counting number greater than 1 that is divisible by a number other than itself and 1. Every composite number has three or more factors. Every composite number can be expressed as a product of two or more prime numbers. *9 is divisible by 1, 3, and 9. It is **composite.*** *11 is divisible by 1 and 11. It is not **composite.***
número compuesto	Número de conteo mayor que 1, divisible entre algún otro número distinto de sí mismo y de 1. Cada número compuesto tiene tres o más divisores. Cada número compuesto puede ser expresado como el producto de dos o más números primos. *9 es divisible entre 1, 3 y 9. Es **compuesto.*** *11 es divisible entre 1 y 11. No es **compuesto.***
cone *(83)*	A three-dimensional solid with one curved surface and one flat, circular surface. The pointed end of a cone is its apex.

cone
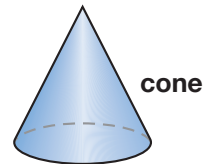
cono	Un sólido tridimensional de base circular y superficie curva. El extremo puntiagudo de un *cono* es su ápice.
congruent *(32)*	Having the same size and shape.
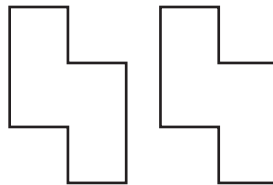
*These polygons are **congruent.** They have the same size and shape.*

congruentes	Que tienen igual tamaño y forma. *Estos polígonos son **congruentes.** Tienen igual tamaño y forma.*

| **continuous data** | Data that can be measured on a scale, such as length, elapsed |
| (Inv. 6) | time, temperature, and cost. |

*Line graphs often display **continuous data.***

datos continuos   Datos que se pueden medir en una escala, tal como longitud, tiempo transcurrido, temperatura y precio.

*Las gráficas lineales frecuentemente muestran **datos continuos.***

---

coordinate(s)
(Inv. 8)

1. A number used to locate a point on a number line.

*The **coordinate** of point A is −2.*

2. A pair of numbers used to locate a point on a coordinate plane.
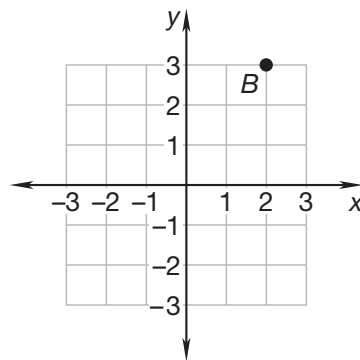
*The **coordinates** of point B are (2, 3). The x-coordinate is listed first, and the y-coordinate is listed second.*

coordenada(s)

1. Número que se utiliza para ubicar un punto sobre una recta numérica.

*La **coordenada** del punto A es –2.*

2. Par ordenado de números que se utiliza para ubicar un punto sobre un plano coordenado.

*Las **coordenadas** del punto B son (2, 3). La coordenada x se escribe primero, seguida de la coordenada y.*

---

coordinate plane
(Inv. 8)

A grid on which any point can be identified by its distances from the *x*- and *y*-axes.
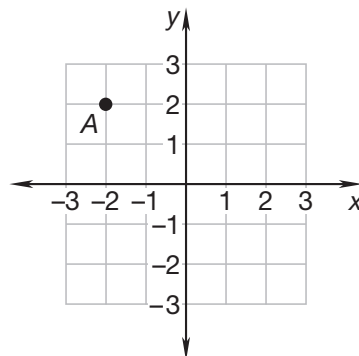
*Point A is located at (−2, 2) on this **coordinate plane.***

plano coordenado   Cuadrícula en que cualquier punto se puede identificar por sus distancias a los ejes *x* e *y*.

*El punto A está ubicado en la posición (−2, 2) sobre este **plano coordenado.***

counting numbers *(1)*	The numbers used to count; the numbers in this sequence: 1, 2, 3, 4, 5, 6, 7, 8, 9, ...
	*The numbers 12 and 37 are **counting numbers,** but 0.98 and $\frac{1}{2}$ are not.*
números de conteo	Números que se utilizan para contar; los números en esta secuencia: 1, 2, 3, 4, 5, 6, 7, 8, 9, ...
	*Los números 12 y 37 son **números de conteo** pero 0.98 y $\frac{1}{2}$ no lo son.*

cube *(83)*	A three-dimensional solid with six square faces. Adjacent faces are perpendicular and opposite faces are parallel.
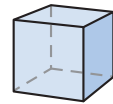
cube

cubo	Un sólido tridimensional con seis caras cuadradas. Las caras adyacentes son perpendiculares y las caras opuestas son paralelas.

cubic unit *(103)*	A cube with edges of designated length. Cubic units are used to measure volume.
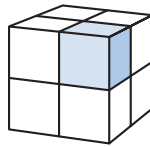
*The shaded part is 1 **cubic unit.** The volume of the large cube is 8 **cubic units.***

unidad cúbica	Un cubo con aristas de una longitud designada. Las unidades cúbicas se usan para medir volumen.
	*La parte sombreada tiene 1 **unidad cúbica.** El volumen del cubo mayor es de 8 **unidades cúbicas.***

cylinder *(83)*	A three-dimensional solid with two circular bases that are opposite and parallel to each other.
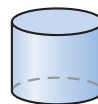
cylinder

cilindro	Un sólido tridimensional con dos bases circulares que son opuestas y paralelas entre sí.

D

data *(Inv. 5)*	(Singular: *datum*) Information gathered from observations or calculations.

82, 76, 95, 98, 97, 93

*These **data** are the average daily temperatures for one week in Utah.*

datos	Información reunida de observaciones o cálculos.
	*Estos **datos** son el promedio diario de las temperaturas de una semana en Utah.*

decade (28)	A period of ten years. *The years 2001–2010 make up one **decade.***
década	Un periodo de diez años. *Los años 2001–2010 forman una **década.***
decimal number (64)	A numeral that contains a decimal point. *23.94 is a **decimal number** because it contains a decimal point.*
número decimal	Número que contiene un punto decimal. *23.94 es un **número decimal,** porque tiene punto decimal.*
decimal place(s) (64)	Places to the right of the decimal point. *5.47 has two **decimal places.*** *6.3 has one **decimal place.*** *8 has no **decimal places.***
cifras decimales	Números ubicados a la derecha del punto decimal. *5.47 tiene dos **cifras decimales.*** *6.3 tiene una **cifra decimal.*** *8 no tiene **cifras decimales.***
decimal point (5)	A symbol used to separate the ones place from the tenths place in decimal numbers (or dollars from cents in money). *34.15* ↑ **decimal point**
punto decimal	Símbolo que se usa en números decimales para separar el lugar de las unidades del lugar de décimas.
decimeter (65)	A metric unit of measurement equal to one tenth of a meter.
decímetro	Una unidad de medida métrica igual a una décima de un metro.

degree (°)
(27, Inv. 10)

1. A unit for measuring temperature.
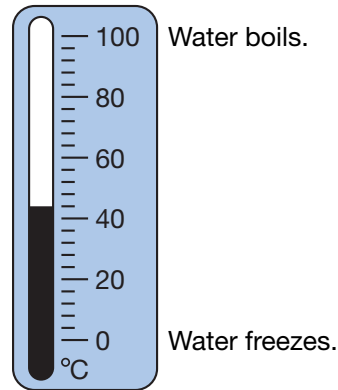
100 — Water boils.	
80	
60	*There are 100 **degrees** (100°) between the freezing and boiling points of water on the Celsius scale.*
40	
20	
0 — Water freezes. °C	

2. A unit for measuring angles.

90°
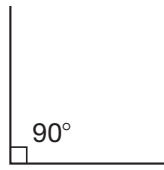
*There are 90 **degrees** (90°) in a right angle.*

360°
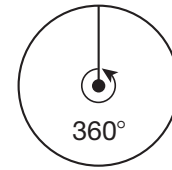
*There are 360 **degrees** (360°) in a circle.*

grado (°)

1. Unidad para medir temperaturas.

Hay 100 **grados** de diferencia entre los puntos de ebullición y congelación del agua en la escala Celsius, o escala centígrada.

2. Unidad para medir ángulos.

Hay 90 grados (90°) en un ángulo recto.

Hay 360 grados (360°) en un círculo.

---

denominator
(Inv. 2)

The bottom number of a fraction; the number that tells how many parts are in a whole.

$\frac{1}{4}$
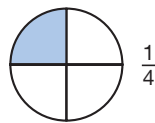
*The **denominator** of the fraction is 4. There are 4 parts in the whole circle.*

denominador

El número inferior de una fracción; el número que indica cuántas partes hay en un todo.

El **denominador** de la fracción es 4. Hay 4 partes en el círculo completo.

---

diameter
(53)

The distance across a circle through its center.

3 in.
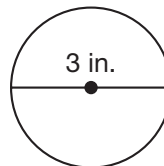
*The **diameter** of this circle is 3 inches.*

diámetro

Distancia entre dos puntos opuestos de un círculo a través de su centro.

El **diámetro** de este círculo mide 3 pulgadas.

difference *(8)*	The result of subtraction. $12 - 8 = 4$ *The **difference** in this problem is 4.*
diferencia	Resultado de una resta. *La **diferencia** en este problema es 4.*
digit *(1)*	Any of the symbols used to write numbers: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9. *The last **digit** in the number 7862 is 2.*
dígito	Cualquiera de los símbolos que se utilizan para escribir números: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9. *El último **dígito** del número 7862 es 2.*
dimension *(72)*	The perpendicular measures of a figure. *Length and width are **dimensions** of a rectangle. Length, width, and height are **dimensions** of a rectangular prism.*
dimensión	Las medidas perpendiculares de una figura. *Largo y ancho son **dimensiones** de un rectángulo. Largo, ancho y altura son **dimensiones** de un prisma rectangular.*
Distributive Property *(51)*	A number times the sum of two addends is equal to the sum of that same number times each individual addend: $$a \times (b + c) = (a \times b) + (a \times c)$$ $$8 \times (2 + 3) = (8 \times 2) + (8 \times 3)$$ *Multiplication is **distributive** over addition.*
propiedad distributiva	Un número multiplicado por la suma de dos sumandos es igual a la suma de los productos de ese número por cada uno de los sumandos. *La multiplicación es **distributiva** con respecto a la suma.*
dividend *(20)*	A number that is divided. $12 \div 3 = 4$ $3\overline{)12}$ $\dfrac{12}{3} = 4$ *The **dividend** is 12 in each of these problems.*
dividendo	Número que se divide en una división. *El **dividendo** es 12 en cada una de estas operaciones.*
divisibility *(42)*	The ability for a number to be divided by another number without a remainder.
divisibilidad	La característica de un número de ser dividido entre otro número sin dar residuo.
divisible *(25)*	Able to be divided by a whole number without a remainder. $4\overline{)20}$ (5) *The number 20 is **divisible** by 4, since 20 ÷ 4 has no remainder.* $3\overline{)20}$ (6 R 2) *The number 20 is not **divisible** by 3, since 20 ÷ 3 has a remainder.*

divisible	Número que se puede dividir sin residuo entre un entero.
	*El número 20 es **divisible** entre 4, ya que no tiene residuo.*
	*El número 20 no es **divisible** entre 3, ya que tiene residuo.*

division
(19)

An operation that separates a number into a given number of equal parts or into a number of parts of a given size.

$21 \div 3 = 7$ *We use **division** to separate 21 into 3 groups of 7.*

división
Una operación que separa un número en un número dado de partes iguales o en un número de partes de una medida dada.

*Usamos la **división** para separar 21 en 3 grupos de 7.*

divisor
(20)

A number by which another number is divided.

$$12 \div 3 = 4 \quad 3\overline{)12}\,^{4} \quad \frac{12}{3} = 4$$ *The **divisor** is 3 in each of these problems.*

divisor
Número que divide a otro en una división.

*El **divisor** es 3 en cada una de estas operaciones.*

E

edge
(83)

A line segment formed where two faces of a solid intersect.
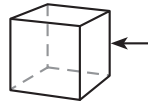
*The arrow is pointing to one **edge** of this cube. A cube has 12 **edges**.*

arista
Un segmento de recta que se forma donde se intersecan dos caras de un sólido.

*La flecha apunta hacia una **arista** de este cubo. Un cubo tiene 12 **aristas.***

elapsed time
(28, 35)

The difference between a starting time and an ending time.

*The race started at 6:30 p.m. and finished at 9:12 p.m. The **elapsed time** of the race was 2 hours 42 minutes.*

tiempo transcurrido
La diferencia entre el tiempo de comienzo y tiempo final.

*La carrera comenzó a las 6:30 p.m. y terminó a las 9:12 p.m . El **tiempo transcurrido** de la carrera fue de 2 horas 42 minutos.*

endpoint(s)
(12, 61)

The points at which a line segment ends.

*Points A and B are the **endpoints** of line segment AB.*

extremo(s)
Punto donde termina un segmento de recta.

*Los puntos A y B son los **extremos** del segmento AB.*

equation *(10)*	A number sentence that uses the equal sign ($=$) to show that two quantities are equal.

$$x = 3 \qquad 3 + 7 = 10 \qquad\qquad 4 + 1 \qquad x < 7$$

equations   not **equations**

ecuación — Enunciado de números que usa el símbolo de igualdad (=) para indicar que dos cantidades son iguales.

---

equilateral triangle
(36)

A triangle in which all sides are the same length and all angles are the same measure.
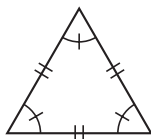
*This is an **equilateral triangle.**
All of its sides are the same length.
All of its angles are the same measure.*

triángulo equilátero — Triángulo que tiene todos sus lados de la misma longitud y todos sus ángulos de la misma medida.

*Éste es un **triángulo equilátero.** Todos sus lados tienen la misma longitud. Todos sus ángulos tienen la misma medida.*

---

equivalent fractions
(79)

Different fractions that name the same amount.

$\frac{1}{2}$ and $\frac{2}{4}$ are **equivalent fractions.**

fracciones equivalentes — Fracciones diferentes que representan la misma cantidad.

$\frac{1}{2}$ y $\frac{2}{4}$ son **fracciones equivalentes.**

---

estimate
(33)

To find an approximate value.

*I **estimate** that the sum of 199 and 205 is about 400.*

estimar — Encontrar un valor aproximado.

*Puedo **estimar** que la suma de 199 más 205 es aproximadamente 400.*

---

evaluate
(78)

To find the value of an expression.

*To **evaluate** $a + b$ for $a = 7$ and $b = 13$, we replace a with 7 and b with 13:*

$$7 + 13 = 20$$

evaluar — Calcular el valor de una expresión.

*Para **evaluar** $a + b$, con $a = 7$ y $b = 13$, se reemplaza a por 7 y b por 13:*

$$7 + 13 = 20$$

---

even numbers
(2)

Numbers that can be divided by 2 without a remainder; the numbers in this sequence: 0, 2, 4, 6, 8, 10, ...

***Even numbers** have 0, 2, 4, 6, or 8 in the ones place.*

números pares — Números que se pueden dividir entre 2 sin residuo; los números en esta secuencia: 0, 2, 4, 6, 8, 10, ...

*Los **números pares** tienen 0, 2, 4, 6 u 8 en el lugar de las unidades.*

event
(57)

An outcome or group of outcomes in an experiment involving probability.

*The **event** of rolling a 4 with one roll of a standard number cube has a probability of $\frac{1}{6}$.*

suceso

El resultado o grupo de resultados en un experimento que involucra probabilidad.

*El **suceso** de obtener un 4 al lanzar una vez un cubo de números tiene una probabilidad de $\frac{1}{6}$.*

---

expanded form
(3)

A way of writing a number that shows the value of each digit.

*The **expanded form** of 234 is 200 + 30 + 4.*

forma desarrollada

Una forma de escribir un número que muestra el valor de cada dígito.

*La **forma desarrollada** de 234 es 200 + 30 + 4.*

---

expanded notation
(48)

A way of writing a number as the sum of the products of the digits and the place values of the digits.

*In **expanded notation** 6753 is written as*

(6 × 1000) + (7 × 100) + (5 × 10) + (3 × 1)

notación desarrollada

Manera de escribir un número como la suma de los productos de cada uno de sus dígitos por su valor de posición.

*En **notación desarrollada** 6753 se escribe como*

(6 × 1000) + (7 × 100) + (5 × 10) + (3 × 1)

---

experiment
(57)

A test to find or illustrate a rule.

*Flipping a coin and selecting an object from a collection of objects are two **experiments** that involve probability.*

experimento

Una prueba para encontrar o ilustrar una regla.

*Lanzar una moneda y seleccionar un objeto de una colección de objetos son dos **experimentos** que involucran probabilidad.*

---

exponent
(78)

The upper number in an exponential expression; it shows how many times the base is to be used as a factor.

$$base \longrightarrow 5^3 \longleftarrow exponent$$

5^3 means 5 × 5 × 5, and its value is 125.

exponente

El número superior en una expresión exponencial; muestra cuántas veces debe usarse la base como factor.

5^3 significa 5 × 5 × 5, y su valor es 125.

---

exponential expression
(78)

An expression that indicates that the base is to be used as a factor the number of times shown by the exponent.

$$4^3 = 4 \times 4 \times 4 = 64$$

*The **exponential expression** 4^3 uses 4 as a factor 3 times. Its value is 64.*

expresión exponencial

Expresión que indica que la base debe usarse como factor el número de veces que indica el exponente.

*La **expresión exponencial** se calcula usando 3 veces el 4 como factor. Su valor es 64.*

expression
(18)

A number, a letter, or a combination of both. *Expressions* do not include comparison symbols, such as an equal sign.

*3n is an **expression** that can also be written as 3 × n.*

expresión

Un número, una letra o una combinación de los dos. Las expresiones no incluyen símbolos de comparación, como el signo de igual.

*3n es una **expresión** que también puede ser escrita como 3 × n.*

F

face
(83)

A flat surface of a geometric solid.
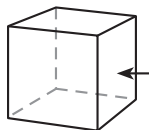
*The arrow is pointing to one **face** of the cube. A cube has six **faces.***

cara

Superficie plana de un cuerpo geométric,.

*La flecha apunta a una **cara** del cubo. Un cubo tiene seis **caras.***

fact family
(8)

A group of three numbers related by addition and subtraction or by multiplication and division.

*The numbers 3, 4, and 7 are a **fact family.** They make these four facts:*

$$3 + 4 = 7 \qquad 4 + 3 = 7 \qquad 7 - 3 = 4 \qquad 7 - 4 = 3$$

familia de operaciones

Grupo de tres números relacionados por sumas y restas o por multiplicaciones y divisiones.

*Los números 3, 4 y 7 forman una **familia de operaciones.** Con ellos se pueden formar estas cuatro operaciones:*

$$3 + 4 = 7 \qquad 4 + 3 = 7 \qquad 7 - 3 = 4 \qquad 7 - 4 = 3$$

factor
(15, 25)

1. Noun: Any one of the numbers multiplied in a multiplication problem.

$2 × 3 = 6$ *The **factors** in this problem are 2 and 3.*

2. Noun: A whole number that divides another whole number without a remainder.

*The numbers 2 and 3 are **factors** of 6.*

3. Verb: To write as a product of ***factors.***

*We can **factor** the number 6 by writing it as 2 × 3.*

factor (n); factorizar (v)

1. Nombre o sustantivo: Cualquiera de los números multiplicados en un problema de multiplicación.

$2 × 3 = 6$ *Los **factores** en esta operación son el 2 y el 3.*

2. Nombre o sustantivo: Número entero que divide a otro número entero sin residuo.

*Los números 2 y 3 son **factores** de 6.*

3. Verbo: Escribir como producto de ***factores.***

*Se puede **factorizar** el número 6 escribiéndolo como el producto 2 × 3.*

Fahrenheit (27)	A scale used on some thermometers to measure temperature. *On the **Fahrenheit** scale, water freezes at 32°F and boils at 212°F.*
Fahrenheit	Escala que se usa en algunos termómetros para medir temperatura. *En la escala **Fahrenheit,** el agua se congela a 32°F y hierve a 212°F.*
Fibonacci sequence (Inv. 4)	A famous sequence in mathematics, which follows an addition pattern. *1, 1, 2, 3, 5, 8, …* *Each term equals the sum of the two terms before it.* *1 + 1 = 2, 1 + 2 = 3, 2 + 3 = 5 …*
secuencia de Fibonacci	Una famosa secuencia matemática que sigue un patrón de suma. *1, 1, 2, 3, 5, 8 …* *Cada término es igual a la suma de los dos términos anteriores.* *1 + 1 = 2, 1 + 2 = 3, 2 + 3 = 5 …*
fluid ounce (85)	A unit of liquid measurement in the customary system equal to one sixteenth of a pint.
onza líquida	Una unidad de medida para líquidos en el sistema usual que es igual a un dieciseisavo de pinta.
formula (11, 72)	An expression or equation that describes a method for solving a certain type of problem. We often write *formulas* with letters that stand for complete words. *A **formula** for the perimeter of a rectangle is P = 2l + 2w, where P stands for "perimeter," l stands for "length," and w stands for "width."*
fórmula	Una expresión o ecuación que describe un método para resolver cierto tipo de problemas. Frecuentemente escribimos *fórmulas* con letras que representan palabras completas. *Una **fórmula** para el perímetro del rectángulo es P = 2l + 2w, donde P representa "perímetro", l representa "longitud" y w representa "ancho".*
fraction (Inv. 2)	A number that names part of a whole. $\frac{1}{4}$ *of the circle is shaded.* $\frac{1}{4}$ *is a **fraction.***
fracción	Número que representa una parte de un entero. $\frac{1}{4}$ *del círculo está sombreado.* $\frac{1}{4}$ *es una **fracción.***
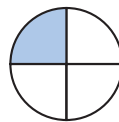
frequency
(Inv. 5)

The number of times an event or outcome occurs.

Purchased Lunch

Number of Lunches	Tally	Frequency
0		0
1	|	1
2	||||	4
3	⫴⫴ ||	7
4	⫴⫴ ⫴⫴	10
5	|||	3

*This table shows the **frequency** of students that purchased lunch.*

frecuencia

El número de veces que un suceso o resultado ocurre.

*Esta tabla muestra la **frecuencia** de los estudiantes que compraron almuerzo.*

frequency table
(Inv. 5)

A table that is used to tally and display the number of times an event or outcome occurs.

Race Results

Laps Completed	Tally	Frequency
0		0
1	|	1
2	||||	4
3	⫴⫴ ||	7
4	⫴⫴ ⫴⫴	10
5	|||	3

*This **frequency table** summarizes the students' performance at the race.*

tabla de frecuencias

Una tabla que se utiliza para tabular y mostrar el número de veces que un suceso o resultado ocurre.

*Esta **tabla de frecuencias** resume el desempeño de los estudiantes en la competencia.*

function table
(Inv. 4)

A table that shows the relationship (or function) between related pairs of numbers.

In	Out
3	6
4	8
7	14

*This **function table** uses the rule "multiply by two."*

| **tabla de función** | Una tabla que muestra la relación (o función) entre pares de números relacionados. |
| | *Esta **tabla de función** utiliza la regla "multiplicar por dos".* |

G

| **geometric sequence** *(Inv. 4)* | A sequence in which each term is found by multiplying the previous term by a fixed amount. |

$$\times 3 \quad \times 3 \quad \times 3 \quad \times 3$$
$$1, \quad 3, \quad 9, \quad 27, \quad 81, \ldots$$

*We multiply a term by 3 to find the term that follows it in this **geometric sequence.***

| **progresión geométrica** | Una secuencia en la que cada término se encuentra multiplicando el término anterior por una cantidad fija. |

$$\times 3 \quad \times 3 \quad \times 3 \quad \times 3$$
$$1, \quad 3, \quad 9, \quad 27, \quad 81, \ldots$$

*Multiplicamos un término por 3 para encontrar el término que sigue en esta **progresión geométrica.***

| **geometric solid** *(83)* | A shape that takes up space. |
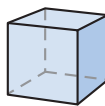
geometric solids | not **geometric solids**
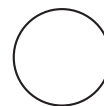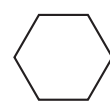
cube   cylinder   circle   rectangle   hexagon

| **sólido geométrico** | Figura que ocupa un espacio. |

| **geometry** *(12)* | A major branch of mathematics that deals with shapes, sizes, and other properties of figures. |

*Some of the figures we study in **geometry** are angles, circles, and polygons.*

| **geometría** | Una rama principal de matemáticas que trata de las formas, tamaños y otras propiedades de figuras. |

*Algunas de las figuras que se estudian en **geometría** son los ángulos, círculos y polígonos.*

| **graph** *(Inv. 5)* | **1.** Noun: A diagram that shows data in an organized way. *See also* **bar graph, circle graph, line graph,** *and* **pictograph.** |

Rainy Days

Days: 8, 6, 4, 2

Jan.   Feb.   Mar.   Apr.

bar **graph**

Shoe Colors of Students

Brown 4, Red 2, Blue 4, Black 6
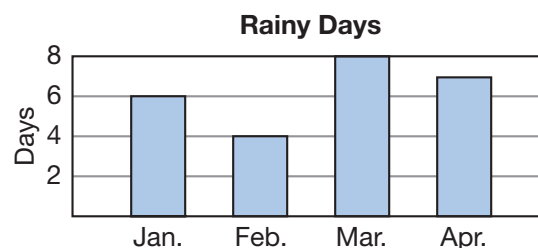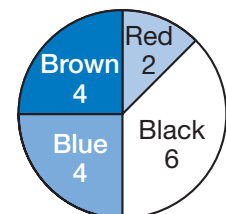
circle **graph**

2. Verb: To draw a point, line, or curve on a coordinate plane.

| **gráfica (n); graficar (v)** | **1.** Nombre: Un diagrama que muestra datos de una forma organizada. *Ver también* **gráfica de barras, gráfica circular, gráfica lineal** *y* **pictograma.** |
| | **2.** Verbo: Dibujar un punto, línea o curva en un plano coordenado. |

greatest common factor (GCF) *(82)*	The largest whole number that is a factor of two or more given numbers.

The factors of 20 are 1, 2, 4, 5, 10, and 20.

The factors of 30 are 1, 2, 3, 5, 6, 10, 15, and 30.

The common factors of 20 and 30 are 1, 2, 5, and 10.

*The **greatest common factor** of 20 and 30 is 10.* |
| **máximo común divisor (MCD)** | Es el mayor número entero que es factor de dos o más números.

Los factores de 20 son 1, 2, 4, 5, 10 y 20.

Los factores de 30 son 1, 2, 3, 5, 6, 10, 15 y 30.

*El **máximo común divisor** de 20 y 30 es 10.* |

H

half *(2)*	One of two equal parts that together equal a whole.
mitad	Una de dos partes que juntas forman un entero.

hexagon *(32)*	A six-sided polygon.

he agon |
| **hexágono** | Un polígono de seis lados. |

histogram *(Inv. 7)*	A method of displaying a range of data. A histogram is a special type of bar graph that displays data in intervals of equal size with no space between bars.

Student Reading Time
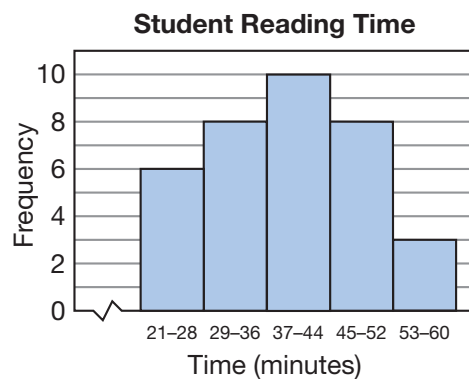
This is a **histogram.** |
| **histograma** | Método para representar un conjunto de datos. Un histograma es un tipo especial de gráfica de barras que muestra los datos a intervalos de igual tamaño y de manera continua sin espacios entre las barras. |

horizontal *(12)*	Side to side; perpendicular to vertical.

oblique line   vertical line
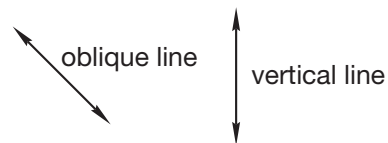
horizontal line   not **horizontal** lines |
| **horizontal** | Lado a lado; perpendicular a una vertical. |

horizontal axis
(Inv. 6)

The scale of a graph that runs from left to right.

eje horizontal

La escala de una gráfica que va de izquierda a derecha.

I

---

icon
(Inv. 7)

A symbol used in a pictograph to represent data.

Consumed by Matt in One Day	
Water	🥤 🥤 🥤 🥤 🥤 🥤
Soda	🥤
Milk	🥤 🥤 🥤 🥤
Juice	🥤 🥤 🥤

Key: 🥤 = 1 cup = 8 ounces

*Each **icon** in the pictograph represents 1 cup of liquid that was consumed.*

icono

Un símbolo que se usa en un pictograma para representar datos.

*Cada **icono** en el pictograma representa 1 taza de líquido consumida.*

Identity Property of Addition
(6)

The sum of any number and 0 is equal to the initial number. In symbolic form, $a + 0 = a$. The number 0 is referred to as the *additive identity*.

*The **Identity Property of Addition** is shown by this statement:*

$$13 + 0 = 13$$

propiedad de identidad de la suma

La suma de cualquier número más 0 es igual al número inicial. En forma simbólica, $a + 0 = a$. El 0 se conoce como *identidad aditiva*.

*La **propiedad de identidad de la suma** se muestra en el siguiente enunciado:*

$$13 + 0 = 13$$

Identity Property of Multiplication
(15)

The product of any number and 1 is equal to the initial number. In symbolic form, $a \times 1 = a$. The number 1 is referred to as the *multiplicative identity*.

*The **Identity Property of Multiplication** is shown by this statement:*

$$94 \times 1 = 94$$

propiedad de identidad de la multiplicación

El producto de cualquier número por 1 es igual al número inicial. En forma simbólica, $a \times 1 = a$. El número 1 se conoce como *identidad multiplicativa*.

*La **propiedad de identidad de la multiplicación** se muestra en el siguiente enunciado:*

$$94 \times 1 = 94$$

impossible *(57)*	We say that an event is *impossible* when the event's probability is 0. This means the event will definitely not occur.
imposible	Decimos que un suceso es *imposible* cuando la probabilidad de que el suceso ocurra es 0. Esto significa que el suceso definitivamente no ocurrirá.
improper fraction *(75)*	A fraction with a numerator greater than or equal to the denominator. $\dfrac{3}{4}$ $\dfrac{2}{2}$ *These fractions are **improper fractions.***
fracción impropia	Fracción con el numerador igual o mayor que el denominador. $\dfrac{3}{4}$ $\dfrac{2}{2}$ *Estas fracciones son **fracciones impropias.***
integers *(12)*	The set of counting numbers, their opposites, and zero; the members of the set $\{\ldots, -2, -1, 0, 1, 2, \ldots\}$. *−57 and 4 are **integers.** $\frac{15}{8}$ and −0.98 are not **integers.***
enteros positivos, negativos y el cero	Conjunto de números de conteo, sus opuestos y el cero; los elementos del conjunto $\{\ldots, -2, -1, 0, 1, 2, \ldots\}$. *−57 y 4 son **enteros.** $\frac{15}{8}$ y −0.98 no son **enteros.***
International System of Units *(44)*	*See **metric system.***
Sistema internacional de unidades	*Ver **sistema métrico.***
intersect *(31)*	To share a common point or points. *These two lines **intersect.** They share the common point M.*
intersecar	Tener uno o más puntos en común. *Estas dos rectas se **intersecan.** Tienen el punto común M.*
intersecting lines *(31)*	Lines that cross. **intersecting lines**
líneas que se intersecan	Líneas que se cruzan.
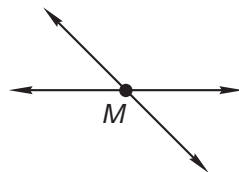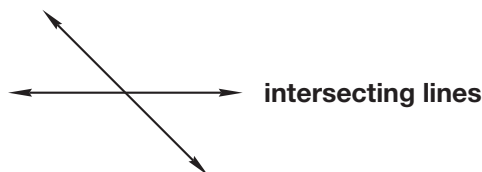
inverse operation(s) (8)	An operation that undoes another.

$a + b - b = a$ *Addition is the **inverse operation***

$a - b + b = a$ *of subtraction.*

$a \times b \div b = a$ $(b \neq 0)$ *Multiplication and division are*

$a \div b \times b = a$ $(b \neq 0)$ ***inverse operations.***

$\sqrt{a^2} = a$ $(a \geq 0)$ *Squaring and finding square*

$(\sqrt{a})^2 = a$ $(a \geq 0)$ *roots are **inverse operations.***

operación inversa — Una operación que cancela a otra.

$a + b - b = a$ *La suma es la **operación inversa** de la resta.*
$a - b + b = a$

$a \times b \div b = a$ $(b \neq 0)$ *La multiplicación y la división son **operaciones**
$a \div b \times b = a$ $(b \neq 0)$ **inversas.***

$\sqrt{a^2} = a$ $(a \geq 0)$ *Elevar a una potencia y calcular la raíz cuadrada son
$(\sqrt{a})^2 = a$ $(a \geq 0)$ **operaciones inversas.***

---

invert (95) — To switch the numerator and denominator of a fraction to form its reciprocal.

*If we **invert** the fraction $\frac{3}{4}$, we get $\frac{4}{3}$.*

invertir — Intercambiar el numerador y el denominador en una fracción para formar su recíproco.

*Al **invertir** la fracción $\frac{3}{4}$, se obtiene $\frac{4}{3}$.*

---

isosceles triangle (36) — A triangle with at least two sides of equal length and two angles of equal measure.
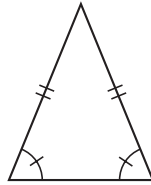
*Two of the sides of this **isosceles triangle** have equal lengths. Two of the angles have equal measures.*

triángulo isósceles — Triángulo que tiene por lo menos dos lados de igual longitud y dos ángulos de igual medida.

*Dos de los lados de este **triángulo isósceles** tienen igual longitud. Dos de los ángulos tienen igual medida.*

---

itinerary (108) — A type of schedule that lists locations and destinations together with departure and arrival times.

Mr. Jones' Trip

Arrival City	Time	Departure City	Time
		Boston, MA	6:55 a.m.
New York City, NY	7:40 a.m.	Portland, ME	9:28 a.m.
Portland, ME	10:51 a.m.	New York City, NY	12:42 p.m.
Boston, MA	1:37 p.m.		

itinerario — Un tipo de programa que enlista el lugar y el destino junto con el tiempo de salida y llegada.

K

kilometer
(74)

A metric unit of length equal to 1000 meters.

*One **kilometer** is approximately 0.62 mile.*

kilómetro

Una unidad métrica de longitud igual a 1000 metros.

*Un **kilometro** es aproximadamente 0.62 milla.*

L

leap year
(28)

A year with 366 days; not a common year.

*A **leap year** occurs every year that is divisible by 4, except for century years that are not divisible by 400. Thus the years 1700, 1800, and 1900 are not leap years because they are not divisible by 400 but 2000 is a leap year.*

año bisiesto

Un año con 366 días; no es un año común.

*Un **año bisiesto** ocurre cada año que es divisible entre 4, a excepción de siglos que no son divisibles entre 400. Por lo tanto, los años 1700, 1800 y 1900 no fueron años bisiestos porque no son divisibles entre 400 pero el 200 fue un año bisiesto.*

least common multiple (LCM)
(112)

The smallest whole number that is a multiple of two or more given numbers.

The multiples of 4 are 4, 8, 12, 16, 20, …

The multiples of 6 are 6, 12, 18, 24, 30, …

*The **least common multiple** of 4 and 6 is 12.*

mínimo común múltiplo (mcm)

El menor número entero que es múltiplo común de dos o más números dados.

Los múltiplos de 4 son 4, 8, 12, 16, 20, ...
Los múltiplos de 6 son 6, 12, 18, 24, 30, ...
*El **mínimo común múltiplo** de 4 y 6 es 12.*

legend
(Inv. 7)

A notation on a map, graph, or diagram that describes the meaning of the symbols and/or the scale used.

kitchen

living/dining $\frac{1}{4}$ inch = 5 feet

bath
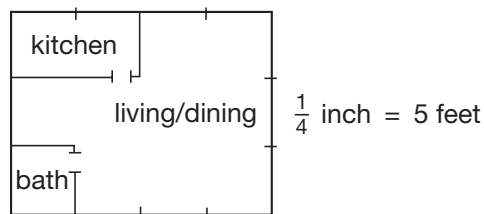
*The **legend** of this scale drawing shows that $\frac{1}{4}$ inch represents 5 feet.*

rótulo

Nota en un mapa, gráfica o diagrama, que describe el significado de los símbolos yo escala usados.

*El **rótulo** de este dibujo a escala muestra que $\frac{1}{4}$ pulg representa 5 pies.*

length
(44)

A measure of the distance between any two points.

3 in.
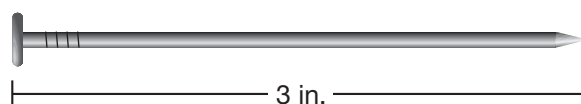
*The **length** of this nail is 3 inches.*

longitud	Una medida de la distancia entre dos puntos. *La **longitud** de este clavo es 3 pulgadas.*

---

line
(12)

A straight collection of points extending in opposite directions without end.

A ●——————————————● B
line *AB* or line *BA*
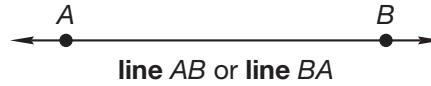
recta Sucesión de puntos que se extiende indefinidamente en ambas direcciones.

---

line graph
(Inv. 5, Inv. 6)

A graph that connects points to show how information changes over time.

Bat Population

Number of Bats (thousands)
40
30
20
10

Time (years)
1 2 3 4

This is a **line graph.**

gráfica lineal Una gráfica que conecta puntos que muestran como cambia la información con el tiempo.

---

line of symmetry
(105)

A line that divides a figure into two halves that are mirror images of each other. *See also* **symmetry.**

lines of symmetry   not **lines of symmetry**

eje de simetría Recta que divide una figura en dos mitades, en la cual una mitad es la imagen espejo de la otra. *Ver **simetría.***

---

line plot
(Inv. 5)

A method of plotting a set of numbers by placing a mark above a number on a number line each time it occurs in the set.

```
                  X
                  X
          X  X  X X        X
   X      X  X X X X X   X X X X
 ◄—┼┼┼┼┼┼┼┼┼┼┼┼┼┼┼┼┼┼┼┼┼┼—►
   0     5      10     15     20
```
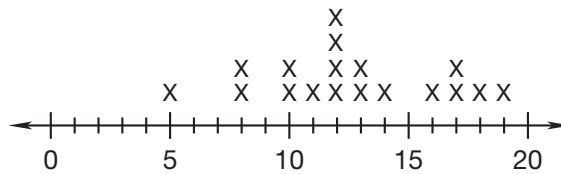
*This is a **line plot** of the numbers 5, 8, 8, 10, 10, 11, 12, 12, 12, 12, 13, 13, 14, 16, 17, 17, 18, and 19.*

diagrama de puntos Método para representar un conjunto de números, que consiste en colocar una marca sobre un número de una recta numérica cada vez que dicho número ocurre en el conjunto.

*Éste es un **diagrama de puntos** de los números 5, 8, 8, 10, 10, 11, 12, 12, 12, 12, 13, 13, 14, 16, 17, 17, 18, y 19.*

line segment (12) **segmento de recta**	A part of a line with two distinct endpoints. *A* *B* $\overline{AB}$ is a **line segment.** Parte de una recta con dos extremos definidos. $\overline{AB}$ *es un* **segmento de recta.**

liter
(85)

A metric unit of capacity or volume.

*A **liter** is a little more than a quart.*

litro

Una unidad métrica de capacidad o volumen.

*Un **litro** es un poco más que un cuarto.*

lowest terms
(81)

A fraction is in *lowest terms* if it cannot be reduced.

*In **lowest terms,** the fraction $\frac{8}{20}$ is $\frac{2}{5}$.*

mínima expresión

Una fracción está en su *mínima expresión* si no se puede reducir.

*Cuando se escribe en su **mínima expresión,** la fracción $\frac{8}{20}$ se convierte en $\frac{2}{5}$.*

M

mass
(77)

The amount of matter an object contains. A kilogram is a metric unit of mass.

*The **mass** of a bowling ball would be the same on the moon as on Earth, even though the weight of the bowling ball would be different.*

masa

Cantidad de materia contenida en un objeto. Un kilogramo es una unidad métrica de masa.

*La **masa** de una bola de boliche es la misma en la Luna que en la Tierra, aunque el peso de la bola de boliche es diferente.*

mean
(84)

See **average.**

media

Ver **promedio.**

measure of central tendency
(84)

A value that describes a property of a list of data, such as the middle number of the list or the number that appears in the list most often. *See also* **mean, median,** *and* **mode.**

1, 3, 5, 6, 8, 9, 13

*The median of this set is 6. The median of a set is one **measure of central tendency.***

medida de tendencia central

Un valor que describe la propiedad de una lista de datos, tal como el número de en medio de la lista o el número que aparece en la lista con mayor frecuencia. *Ver también* **media, mediana** *y* **moda.**

1, 3, 5, 6, 8, 9, 13

*La mediana en este conjunto es 6. La mediana de un conjunto es una **medida de tendencia central.***

median
(Inv. 5, 84)

The middle number (or the average of the two central numbers) of a list of data when the numbers are arranged in order from the least to the greatest.

1, 1, 2, 4, 5, 7, 9, 15, 24, 36, 44

*In this list of data, 7 is the **median.***

mediana

Número de en medio (o el promedio de los dos números centrales) en una lista de datos, cuando los números se ordenan de menor a mayor.

*1, 1, 2, 4, 5, 7, 9, 15, 24, 36, 44   En esta lista de datos, 7 es la **mediana.***

---

meter
(65)

The basic unit of length in the metric system.

*Many classrooms are about 10 **meters** long and 10 **meters** wide.*

metro

La unidad básica de longitud en el sistema métrico.

*Muchos salones de clase miden aproximadamente 10 **metros** de largo por 10 **metros** de ancho.*

---

metric system
(44)

An international system of measurement in which units are related by a power of ten. Also called the *International System.*

*Centimeters and kilograms are units in the **metric system.***

sistema métrico

Un sistema internacional de medición en el cual las unidades de medida se relacionan por potencias de diez. También se le llama *Sistema internacional.*

*Centímetros y kilogramos son unidades del **sistema métrico.***

---

mill
(68)

An amount of money equal to one thousandth of a dollar (one tenth of a penny).

*The gasoline price of $3.199 per gallon equals $3.19 plus 9 **mills.***

milésima

Una cantidad de dinero igual a una milésima de un dólar (un décimo de un centavo).

*El precio de $3.199 por galón es igual a $3.19 más 9 **milésimas.***

---

millennium
(28)

A period of one thousand years.

*The years 2001–3000 make up one **millennium.***

milenio

Un período de mil años.

*Los años 2001–3000 forman un **milenio.***

---

millimeter
(44, 65)

A metric unit of length equal to one thousandth of a meter.

*There are 1000 **millimeters** in 1 meter and 10 **millimeters** in one centimeter.*

milímetro

Una unidad métrica de longitud que es igual a una milésima de un metro.

*Hay 1000 **milímetros** en 1 metro y 10 **milímetros** en un centímetro.*

---

mixed number
(38)

A number expressed as a whole number plus a fraction.

*The **mixed number** $2\frac{1}{3}$ means "two and one third."*

número mixto

Número formado por un número entero y una fracción.

*El **número mixto** $2\frac{1}{3}$ significa "dos y un tercio".*

mode *(Inv. 5, 84)*	The number or numbers that appear most often in a list of data. *5, 12, 32, 5, 16, 5, 7, 12* *In this list of data, the number 5 is the **mode.***
moda	Número o números que aparecen con más frecuencia en una lista de datos. *5, 12, 32, 5, 16, 5, 7, 12* *En esta lista de datos, el número 5 es la **moda.***
multiple *(15, 29)*	A product of a counting number and another number. *The **multiples** of 3 include 3, 6, 9, and 12.*
múltiplo	Producto de un número de conteo por otro número. *Los **múltiplos** de 3 incluyen 3, 6, 9 y 12.*
multiplication table *(15)*	A table used to find the product of two numbers. The product of two numbers is found at the intersection of the row and the column for the two numbers.
tabla de multiplicación	Una tabla que se utiliza para encontrar el producto de dos números. El producto de dos números se encuentra en la intersección de la fila y la columna para los dos números.
mutually exclusive *(Inv. 7)*	Categories are mutually exclusive if each data point can be placed in one, and only one, of the categories. *When flipping one coin, the categories are "landing heads-up" and "landing tails-up." One coin cannot land both heads-up and tails-up on the same toss. Thus, the categories "landing heads-up" and "landing tails-up" are **mutually exclusive.***
mutuamente excluyentes	Dos categorías son mutuamente excluyentes si cada punto de los datos puede ser colocado en una, y solo una, de las categorías. *Cuando se lanza una moneda las categorías son "que caiga cara arriba" y "que caiga cara abajo", una moneda no puede caer cara arriba y cara abajo en el mismo lanzamiento. Por lo tanto, las categorías "que caiga cara arriba" y "que caiga cara abajo" son **mutuamente excluyentes.***

N

negative numbers *(12)*	Numbers less than zero. *-15 and -2.86 are **negative numbers.*** *19 and 0.74 are not **negative numbers.***
números negativos	Los números menores que cero. *-15 y -2.86 son **números negativos.*** *19 y 0.74 no son **números negativos.***
number line *(12)*	A line for representing and graphing numbers. Each point on the line corresponds to a number. number line $-2 \quad -1 \quad 0 \quad 1 \quad 2 \quad 3 \quad 4 \quad 5$
recta numérica	Recta para representar y graficar números. Cada punto de la recta corresponde a un número.

numeral *(Appendix A)*	A symbol or group of symbols that represents a number. *4, 72, and $\frac{1}{2}$ are examples of **numerals.*** *"Four," "seventy-two," and "one half" are words that name numbers but are not **numerals.***
numeral	Símbolo, o grupo de símbolos numéricos, que representa un número. *4, 72 y $\frac{1}{2}$ son ejemplos de **numerales.** "Cuatro", "setenta y dos" y "un medio" son palabras que identifican números, pero no son **numerales.***

numerator *(Inv. 2)*	The top number of a fraction; the number that tells how many parts of a whole are counted. $\frac{1}{4}$ *The **numerator** of the fraction is 1.* *One part of the whole circle is shaded.*
numerador	El término superior de una fracción. El número que nos dice cuántas partes de un entero se cuentan. *El **numerador** de la fracción es 1. Una parte del círculo está sombreada.*
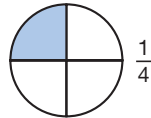
O

oblique *(12, 31)*	**1.** Slanted or sloping: not horizontal or vertical. **oblique** line horizontal line vertical line not **oblique** lines **2.** Lines in the same plane that are neither parallel nor perpendicular. **oblique lines** perpendicular lines parallel lines not **oblique lines**
recta(s) oblicua(s)	**1.** Recta que no es ni horizontal ni vertical. **2.** Rectas ubicadas en un mismo plano, que no son ni paralelas ni perpendiculares entre sí.
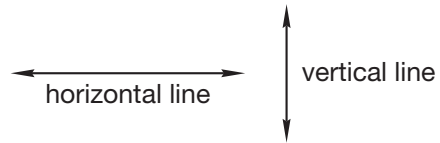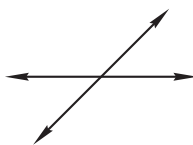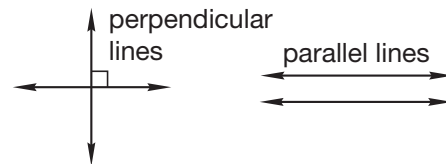
obtuse angle *(31)*	An angle whose measure is more than 90° and less than 180°. **obtuse angle** right angle acute angle not **obtuse angles** *An **obtuse angle** is larger than both a right angle and an acute angle.*
ángulo obtuso	Ángulo que mide más de 90° y menos de 180°. *Un **ángulo obtuso** es más grande que un ángulo recto y que un ángulo agudo.*
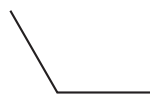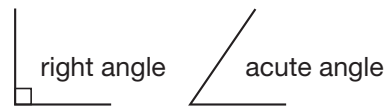
obtuse triangle (36)	A triangle whose largest angle measures more than 90° and less than 180°.
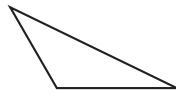
obtuse triangle　　acute triangle　right triangle

not **obtuse triangles**

triángulo obtusángulo	Triángulo cuyo ángulo mayor mide más de 90° y menos de 180°.

---

octagon (32)	A polygon with eight sides.

octagon

octágono	Un polígono con ocho lados.

---

odd numbers (2)	Numbers that have a remainder of 1 when divided by 2; the numbers in this sequence: 1, 3, 5, 7, 9, 11, ...
	***Odd numbers** have 1, 3, 5, 7, or 9 in the ones place.*
números impares	Números que cuando se dividen entre 2 tienen residuo 1; los números en la secuencia: 1, 3, 5, 7, 9, 11, ...
	*Los **números impares** tienen 1, 3, 5, 7 ó 9 en el lugar de las unidades.*

---

operations of arithmetic (24)	The four basic mathematical operations: addition, subtraction, multiplication, and division.

$$1 + 9 \qquad 21 - 8 \qquad 6 \times 22 \qquad 3 \div 1$$

the **operations of arithmetic**

operaciones aritméticas	Las cuatro operaciones matemáticas básicas: suma, resta, multiplicación y división.

---

opposite sides (83)	Sides that are across from each other.

opposite sides
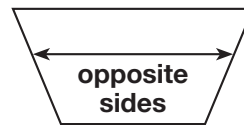
lados opuestos	Lados que están uno enfrente del otro.

---

ordinal numbers (7)	Numbers that describe position or order.
	*"First," "second," and "third" are **ordinal numbers.**_
números ordinales	Números que describen orden o posición.
	*"Primer," "segundo" y "tercero" son **números ordinales.**_

origin
(Inv. 8)

1. The location of the number 0 on a number line.

origin on a number line
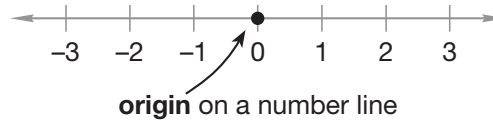
2. The point (0,0) on a coordinate plane.

origin on a
coordinate plane
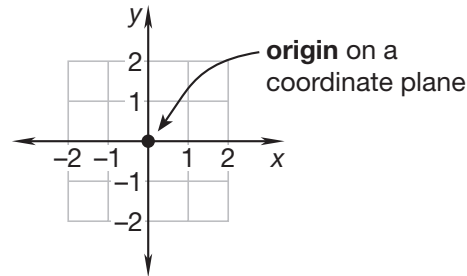
origen

1. Posición del número 0 en una recta numérica.

2. El punto (0, 0) en un plano coordenado.

ounce
(85)

A unit of weight in the customary system. Also a measure of liquid capacity. *See also* **fluid ounce.**

*Sixteen **ounces** equals a pound. Sixteen **fluid ounces** equals a pint.*

onza

Una unidad de peso en el sistema usual. También una medida de capacidad. *Ver también* **onza líquida.**

*Dieciséis **onzas** es igual a una libra. Dieciséis **onzas líquidas** es igual a una pinta.*

outcome
(57)

Any possible result of an experiment.

*When rolling a number cube, the possible **outcomes** are 1, 2, 3, 4, 5, and 6.*

resultado

Cualquier resultado posible de un experimento.

*Cuando se lanza un cubo de números los **resultados** posibles son 1, 2, 3, 4, 5 y 6.*

outlier
(Inv. 5)

A number that is distant from most of the other numbers in a list of data.

*In the data at right, the number 28 is an **outlier** because it is distant from the other numbers in the list.*   1, 5, 4, 3, 6, 28, 7, 2

valor extremo

Número en una lista de datos, que es mucho mayor o mucho menor que los demás números de la lista.

*En los datos a la derecha, el número 28 es un **valor extremo,** porque su valor es mayor que el de los demás números de la lista.*

P

parallel lines
(31)

Lines that stay the same distance apart; lines that do not cross.

parallel lines
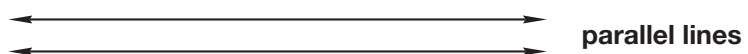
rectas paralelas

Rectas ubicadas en un mismo plano y que nunca se intersecan.

parallelogram *(45)*	A quadrilateral that has two pairs of parallel sides.

parallelograms   not a **parallelogram**

paralelogramo	Cuadrilátero que tiene dos pares de lados paralelos.

---

parentheses
(24)

A pair of symbols used to separate parts of an expression so that those parts may be evaluated first: ( ).

$$15 - (12 - 4)$$

*In the expression 15 − (12 − 4), the **parentheses** indicate that 12 − 4 should be calculated before subtracting the result from 15.*

paréntesis

Un par de símbolos que se utilizan para separar partes de una expresión para que esas partes puedan ser evaluadas primero: ( ).

*En la expresión 15 − (12 − 4) los **paréntesis** indican que 12 − 4 debe ser calculado antes de restar el resultado de 15.*

---

partial product
(51)

When multiplying using pencil and paper, a product resulting from multiplying one factor by one digit of the other factor. The final product is the sum of the shifted partial products.

$$\begin{array}{r} 53 \\ \times\, 26 \\ \hline 318 \\ +\ 106\ \ \\ \hline 1378 \end{array}$$ **partial products**

producto parcial

Cuando se multiplica usando lápiz y papel, el producto resulta de multiplicar un factor por un dígito del otro factor. El producto final es la suma de los productos parciales desplazados.

---

pentagon
(32)

A polygon with five sides.
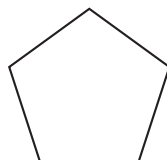
pentagon

pentágono

Un polígono de cinco lados.

---

percent
(30)

A fraction whose denominator of 100 is expressed as a percent sign (%).

$$\frac{99}{100} = 99\% = 99\ \textbf{percent}$$

porcentaje

Fracción cuyo denominador de 100 se expresa con un signo (%), que se lee *por ciento*.

perfect square (78)	The product when a whole number is multiplied by itself. *The number 9 is a **perfect square** because 3 × 3 = 9.*
cuadrado perfecto	Producto cuando un número entero se multiplica por sí mismo. *El número 9 es un **cuadrado perfecto,** porque 3 × 3 = 9.*

---

perimeter
(53)

The distance around a closed, flat shape.
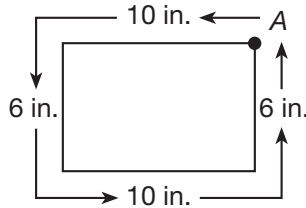
*The **perimeter** of this rectangle (from point A around to point A) is 32 inches.*

perímetro

Distancia alrededor de una figura cerrada y plana.

*El **perímetro** de este rectángulo (desde el punto A alrededor del rectángulo hasta el punto A) es 32 pulgadas.*

---

permutation
(84)

One possible arrangement of a set of objects.

2 4 3 1

*The arrangement above is one possible **permutation** of the numbers 1, 2, 3, and 4.*

permutación

Un arreglo posible de un conjunto de objetos.

*El arreglo anterior es una **permutación** posible de los números 1, 2, 3 y 4.*

---

perpendicular lines
(31)

Two lines that intersect at right angles.

perpendicular lines   not **perpendicular lines**
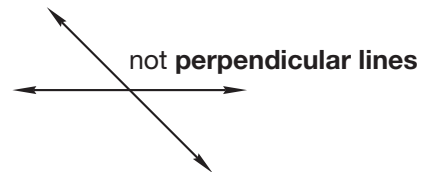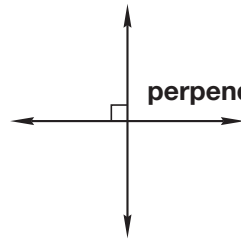
rectas perpendiculares

Dos rectas que se intersecan formando ángulos rectos.

---

pictograph
(Inv. 5, Inv. 7)

A graph that uses symbols to represent data.

*This is a **pictograph.** It shows how many stars each person saw.*

pictograma

Gráfica que utiliza símbolos para representar datos.

*Éste es un **pictograma.** Muestra el número de estrellas que vio cada persona.*

pie chart *(Inv. 7)*	*See* **circle graph.**
diagrama circular	*Vea* **gráfica circular.**
place value *(3)*	The value of a digit based on its position within a number. $\begin{array}{r} 341 \\ 23 \\ +7 \\ \hline 371 \end{array}$ **Place value** tells us that 4 in 341 is worth "4 tens." In addition problems, we align digits with the same **place value.**
valor posicional	Valor de un dígito de acuerdo al lugar que ocupa en el número. *El **valor posicional** indica que el 4 en 341 vale "cuatro decenas". En los problemas de suma y resta, se alinean los dígitos que tienen el mismo **valor posicional.***
plane *(32)*	A flat surface that has no boundaries. *The flat surface of a desk is part of a **plane.***
plano	Superficie plana ilimitada. *La superficie plana de un escritorio es parte de un **plano.***
plane figure *(83)*	A flat shape. **plane figures** not a **plane figure**
figura plana	Una figura plana.
p.m. *(28)*	The period of time from noon to just before midnight. *I go to bed at 9 **p.m.,** which is 9 o'clock at night.*
p.m.	Período de tiempo desde el mediodía hasta justo la medianoche. *Me voy a dormir a las 9 **p.m.** lo cual es las 9 en punto de la noche.*
point *(12, 61)*	An exact position. •*A* This dot represents **point** *A.*
punto	Una posición exacta. *Esta marca representa el **punto** A.*
polygon *(32)*	A closed, flat shape with straight sides. **polygons** not **polygons**
polígono	Figura cerrada y plana que tiene lados rectos.
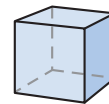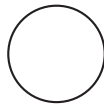
positive numbers
(98)

Numbers greater than zero.

*0.25 and 157 are **positive numbers.***

*−40 and 0 are not **positive numbers.***

número positivos

Números mayores que cero.

*0.25 y 157 son **números positivos.***

*−40 y 0 no son **números positivos.***

power
(78)

1. The value of an exponential expression.

*16 is the fourth **power** of 2 because $2^4 = 16$.*

2. An exponent.

*The expression 2^4 is read "two to the fourth **power.**"*

potencia

1. El valor de una expresión exponencial.

*16 es la cuarta **potencia** de 2, porque $2^4 = 16$.*

2. Un exponente.

*La expresión 2^4 se lee "dos a la cuarta **potencia.**"*

prime number
(25, 80)

A counting number greater than 1 whose only two factors are the number 1 and itself.

*7 is a **prime number.** Its only factors are 1 and 7.*

*10 is not a **prime number.** Its factors are 1, 2, 5, and 10.*

número primo

Número natural mayor que 1, cuyos dos únicos factores son el 1 y el propio número.

*7 es un **número primo.** Sus únicos factores son 1 y 7.*

*10 no es un **número primo.** Sus factores son 1, 2, 5 y 10.*

prism
(89)

A three-dimensional solid with two congruent bases.

prisma

Un sólido tridimensional con dos bases congruentes.

probability
(57)

A way of describing the likelihood of an event; the ratio of favorable outcomes to all possible outcomes.

*The **probability** of rolling a 3 with a standard number cube is $\frac{1}{6}$.*

probabilidad

Manera de describir la ocurrencia de un suceso; la razón de resultados favorables a todos los resultados posibles.

*La **probabilidad** de obtener 3 al lanzar un cubo estándar de números es $\frac{1}{6}$.*

product
(15)

The result of multiplication.

*$5 \times 3 = 15$ The **product** of 5 and 3 is 15.*

producto

Resultado de una multiplicación.

*$5 \times 3 = 15$ El **producto** de 5 por 3 es 15.*

proper fraction (75)	A fraction whose denominator is greater than its numerator. $\frac{3}{4}$ is a **proper fraction.** $\frac{4}{3}$ is not a **proper fraction.**
fracción propia	Una fracción cuyo denominador es mayor que su numerador. $\frac{3}{4}$ es una **fracción propia.** $\frac{4}{3}$ no es una **fracción propia.**
Property of Zero for Multiplication (15)	Zero times any number is zero. In symbolic form, $0 \times a = 0$. *The **Property of Zero for Multiplication** tells us that $89 \times 0 = 0$.*
propiedad del cero en la multiplicación	Cero multiplicado por cualquier número es cero. En forma simbólica, $0 \times a = 0$. *La **propiedad del cero en la multiplicación** dice que $89 \times 0 = 0$.*
protractor (Inv. 10)	A tool used to measure and draw angles. This is a **protractor.**
transportador	Instrumento que sirve para medir y trazar ángulos.
pyramid (83)	A three-dimensional solid with a polygon as its base and triangular faces that meet at a vertex. pyramid
pirámide	Un sólido tridimensional con un polígono en su base y caras triangulares que se encuentran en un vértice.
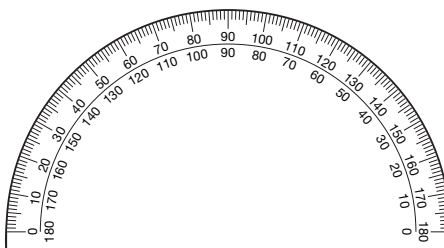
Q

quadrilateral (32)	Any four-sided polygon. *Each of these polygons has 4 sides. They are all **quadrilaterals.***
cuadrilátero	Polígono de cuatro lados. *Cada uno de estos polígonos tiene 4 lados. Todos son **cuadriláteros.***

quotient (20)	The result of division.

$$12 \div 3 = 4 \qquad 3\overline{)12}^{\,4} \qquad \frac{12}{3} = 4$$

*The **quotient** is 4 in each of these problems.*

cociente   Resultado de una división.

*El **cociente** es 4 en cada una de estas operaciones.*

R

radius (53)	(Plural: *radii*) The distance from the center of a circle to a point on the circle.
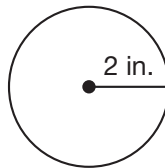
*The **radius** of this circle is 2 inches.*

radio   Distancia desde el centro de un círculo hasta un punto del círculo.

*El **radio** de este círculo es 2 pulgadas.*

range (Inv. 5, 84)	The difference between the largest number and the smallest number in a list.

5, 17, 12, 34, 29, 13

*To calculate the **range** of this list, we subtract the smallest number from the largest number. The **range** of this list is 29.*

intervalo   Diferencia entre el número mayor y el número menor de una lista.

*Para calcular el **intervalo** de esta lista, se resta el número menor del número mayor. El **intervalo** de esta lista es 29.*

ratio (97)	A comparison of two numbers by division.

*There are 3 triangles and 5 stars. The **ratio** of triangles to stars is "three to five," or $\frac{3}{5}$.*

razón   Comparación de dos números por división.

*Hay 3 triángulos y 5 estrellas. La **razón** de triángulos a estrellas es "tres a cinco" ó $\frac{3}{5}$.*

ray (12)	A part of a line that begins at a point and continues without end in one direction.

$$A \bullet\!\!-\!\!-\!\!-\!\!-\!\!-\!\!-\!\!\bullet B \longrightarrow$$

ray *AB* ($\overrightarrow{AB}$)
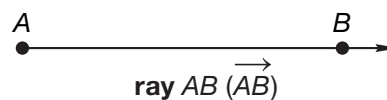
rayo   Parte de una recta que empieza en un punto y continúa indefinidamente en una dirección.

reciprocals *(95)*	Two numbers whose product is 1. $$\frac{3}{4} \times \frac{4}{3} = \frac{12}{12} = 1$$ *Thus, the fractions $\frac{3}{4}$ and $\frac{4}{3}$ are **reciprocals**. The **reciprocal** of $\frac{3}{4}$ is $\frac{4}{3}$.*
recíprocos	Dos números cuyo producto es igual a 1. *Las fracciones $\frac{3}{4}$ y $\frac{4}{3}$ son **recíprocas**. El **recíproco** de $\frac{3}{4}$ es $\frac{4}{3}$.*
rectangle *(45)*	A quadrilateral that has four right angles. **rectangles** not **rectangles**
rectángulo	Cuadrilátero que tiene cuatro ángulos rectos.
rectangular solid *(83)*	A three-dimensional solid having six rectangular faces. Adjacent faces are perpendicular and opposite faces are parallel. **rectangular solid**
sólido rectangular	Un sólido tridimensional que tiene 6 caras rectangulares. Las caras adyacentes son perpendiculares y las caras opuestas son paralelas.
reduce *(81)*	To rewrite a fraction in lowest terms. *If we **reduce** the fraction $\frac{9}{12}$, we get $\frac{3}{4}$.*
reducir	Reescribir o escribir una fracción en su mínima expresión. *Al **reducir** la fracción $\frac{9}{12}$, se obtiene $\frac{3}{4}$.*
reflection *(Inv. 8)*	Flipping a figure to produce a mirror image. **reflection** figure image
reflexión	Inversión de una figura para lograr una imagen especular; imagen reflejada en un espejo.
reflective symmetry *(105)*	A figure has *reflective symmetry* if it can be divided into two halves that are mirror images of each other. *See also* **line of symmetry.** These figures have **reflective symmetry**. These figures do not have **reflective symmetry**.
simetría de reflexión	Una figura tiene *simetría de reflexión* si puede dividirse en dos mitades especulares. *Ver también **eje de simetría.***
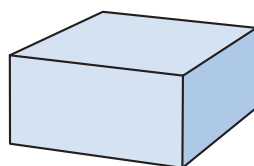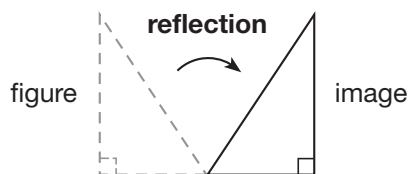
regular polygon	A polygon in which all sides have equal lengths and all angles have equal measures.

(53)
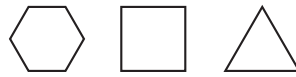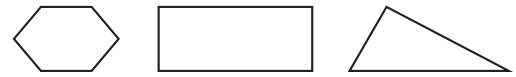
regular polygons   **not regular polygons**

polígono regular   Polígono en el cual todos los lados tienen la misma longitud y todos los ángulos tienen la misma medida.

---

relative frequency table

(Inv. 9)

A frequency table in which the frequencies for all categories are displayed as the numerator of a fraction with the total number of outcomes as the denominator.
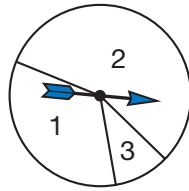
Outcome	Tally	Relative Frequency
1	IIII IIII IIII II	$\frac{17}{50}$
2	IIII IIII IIII IIII IIII III	$\frac{28}{50}$
3	IIII	$\frac{5}{20}$

*This **relative frequency table** shows data obtained by spinning the spinner at left 50 times.*

tabla de frecuencias relativas   Una tabla de frecuencias en donde las frecuencias para todas las categorías se muestran como el numerador de una fracción con el número total de resultados como el denominador.

*Esta **tabla de frecuencias relativas** muestra datos obtenidos al girar la rueda hacia la izquierda 50 veces.*

---

remainder

(22)

An amount left after division.

$$\begin{array}{r} 7\ R\ 1 \\ 2\overline{)15} \\ 14 \\ \hline 1 \end{array}$$

*When 15 is divided by 2, there is a **remainder** of 1.*

residuo   Cantidad que queda después de dividir.

*Cuando se divide 15 entre 2, queda **residuo** 1.*

---

rhombus

(45)

A parallelogram with all four sides of equal length.

rhombuses   **not rhombuses**

rombo   Paralelogramo con sus cuatro lados de igual longitud.

right angle *(31)*	An angle that forms a square corner and measures 90°. It is often marked with a small square.
	right angle obtuse angle / acute angle **right angle** not **right angles**
	*A **right angle** is larger than an acute angle and smaller than an obtuse angle.*
ángulo recto	Ángulo que forma una esquina cuadrada y mide 90°. Frecuentemente se indica con un pequeño cuadrado. *Un **ángulo recto** es mayor que un ángulo agudo y menor que un ángulo obtuso.*
right triangle *(36)*	A triangle whose largest angle measures 90°.
	acute triangle obtuse triangle **right triangle** not **right triangles**
triángulo rectángulo	Triángulo cuyo ángulo mayor mide 90°.
Roman numerals *(Appendix A)*	Symbols used by the ancient Romans to write numbers. *The **Roman numeral** for 3 is III.* *The **Roman numeral** for 13 is XIII.*
números romanos	Símbolos empleados por los antiguos romanos para escribir números. *El **número romano** para 3 es III.* *El **número romano** para 13 es XIII.*
rotation *(Inv. 8)*	Turning a figure about a specified point called the *center of rotation.*
	rotation image figure
rotación	Giro de una figura alrededor de un punto específico llamado *centro de rotación.*
rotational symmetry *(105)*	A figure has *rotational symmetry* if it can be rotated less than a full turn and appear in its original orientation.
	S △ ❃ Z ▱ M ⌂
	These figures have **rotational symmetry.** These figures do not have **rotational symmetry.**
simetría rotacional	Una figura tiene simetría rotacional cuando no requiere de una rotación completa para que la figura aparezca en la misma posición en que comenzó la rotación.
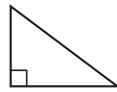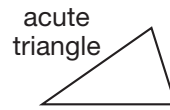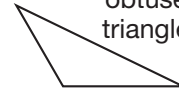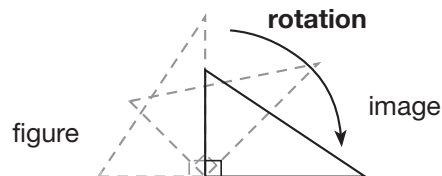
round number *(33)*	A close number to the given number. Rounding a number can help us estimate.
número redondeado	Un número cercano al número dado. Redondear un número nos ayuda a estimar.

S

scale *(27, Inv. 11)*	**1.** A type of number line used for measuring.
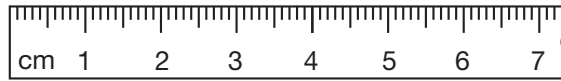
*The distance between each mark on this ruler's **scale** is 1 centimeter.*

2. A ratio that shows the relationship between a scale model and the actual object.

*If a model airplane is $\frac{1}{24}$ the size of the actual airplane, the **scale** of the model is 1 to 24.*

escala	**1.** Un tipo de recta numérica que se utiliza para hacer mediciones. *La distancia entre cada marca en la **escala** de esta regla es 1 centímetro.* **2.** Una razón que nos muestra la relación entre un modelo a escala y el objeto actual. *Si el modelo de un avión es $\frac{1}{24}$ del tamaño del avión real, la **escala** del modelo es 1 a 24.*

scale drawing *(Inv. 11)*	A two-dimensional representation of a larger or smaller object. *Blueprints and maps are examples of **scale drawings.***
dibujo a escala	Representación bidimensional de un objeto más grande o más pequeño. *Los planos y los mapas son ejemplos de **dibujos a escala.***

scale model *(Inv. 11)*	A three-dimensional representation of a larger or smaller object. *Globes and model airplanes are examples of **scale models.***
modelo a escala	Una representación tridimensional de un objeto más pequeño o más grande. *Los globos terrestres y aviones de juguete son ejemplos de **modelos a escala.***

scalene triangle *(36)*	A triangle with three sides of different lengths.
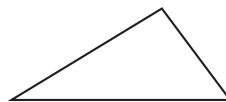
*All three sides of this **scalene triangle** have different lengths.*

triángulo escaleno	Triángulo con todos sus lados de diferente longitud. *Los tres lados de este **triángulo escaleno** tienen diferente longitud.*

schedule (108)	A list of events organized by the times at which they are planned to occur.

Sarah's Class Schedule	
8:15 a.m.	Homeroom
9:00 a.m.	Science
10:15 a.m.	Reading
11:30 a.m.	Lunch and recess
12:15 p.m.	Math
1:30 p.m.	English
2:45 p.m.	Art and music
3:30 p.m.	End of school

calendario, horario   Una lista de sucesos organizada en base a los tiempos en que se planea que dichos sucesos ocurran.

---

sector (57)   A region bordered by part of a circle and two radii.
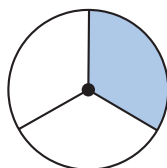
*This circle is divided into 3 **sectors.** One **sector** of the circle is shaded.*

sector   Región de un círculo limitada por un arco y dos radios.

*Este círculo esta dividido en 3 **sectores.** Un **sector** está sombreado.*

---

segment (12)   *See* **line segment.**

segmento   *Ver* **segmento de recta.**

---

sequence (1)   A list of numbers arranged according to a certain rule.

*The numbers 2, 4, 6, 8, … form a **sequence.** The rule is "count up by twos."*

secuencia   Lista de números ordenados de acuerdo a una regla.

*Los números 2, 4, 6, 8, ... forman una **secuencia.** La regla es "contar de dos en dos".*

---

short division (42)   A form of division that differs from long division. In ***short division*** we keep track of some numbers in our head.

división corta   Una forma de división que difiere de una división larga. En la división corta llevamos la cuenta de algunos números mentalmente.

---

side (32)   A line segment that is part of a polygon.
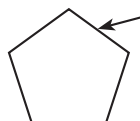
*The arrow is pointing to one **side.** This pentagon has 5 **sides.***

lado	Segmento de recta que forma parte de un polígono.
	*La flecha apunta hacia uno de los **lados**. Este pentágono tiene 5 **lados**.*

---

similar
(32)

Having the same shape but not necessarily the same size. Dimensions of similar figures are proportional.
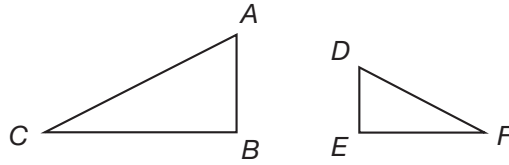
*△ABC and △DEF are **similar**. They have the same shape, but not the same size.*

semejante

Que tiene la misma forma, pero no necesariamente el mismo tamaño. Figuras semejantes son proporcionales.

*△ABC y △DEF son **semejantes**. Tienen la misma forma, pero diferente tamaño.*

---

simplest form
(81)

The form of a fraction when it is reduced to lowest terms.

forma reducida

La forma de una fracción cuando se escribe en su mínima expresión.

---

solid
(83)

*See **geometric solid**.*

sólido

*Ver **sólido geométrico**.*

---

sphere
(83)

A round geometric solid having every point on its surface at an equal distance from its center.

sphere
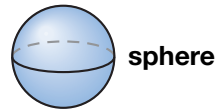
esfera

Un sólido geométrico curvo que tiene cada punto de su superficie a una distancia igual de su centro.

---

spread
(84)

A value that describes how the data in a set are distributed. *See also **range**.*

5, 12, 3, 20, 15

*The range of this set is 17. Range, which is the difference between the greatest and least numbers, is one measure of the **spread** of data.*

extensión

Un valor que describe como los datos en un conjunto son distribuidos. *Ver también **intervalo**.*

*5, 12, 3, 20, 15 El intervalo de este conjunto es 17. El intervalo, que es la diferencia entre el mayor y el menor número es una medida de la **extensión** de los datos.*

square *(45, 78)*	**1.** A rectangle with all four sides of equal length.

<div align="center">

12 mm

12 mm ⬜ 12 mm
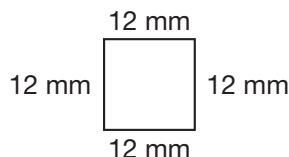
12 mm

*All four sides of this **square** are 12 mm long.*

</div>

2. The product of a number and itself.

*The **square** of 4 is 16.*

cuadrado	**1.** Paralelogramo que tiene cuatro lados de igual longitud. *Los cuatro lados de este **cuadrado** miden 12 mm.* **2.** El producto de un número por sí mismo. *El **cuadrado** de 4 es 16.*

---

square centimeter *(72)*	A measure of area equal to that of a square with 1-centimeter sides.

<div align="center">

1 cm ⬜ **square centimeter**

1 cm

</div>
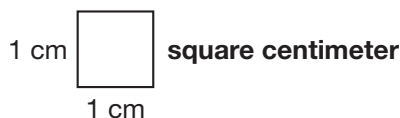
centímetro cuadrado	Medida de un área igual a la de un cuadrado con lados de 1 centímetro.

---

square inch *(72)*	A measure of area equal to that of a square with 1-inch sides.

<div align="center">

1 in. ⬜ **square inch**

1 in.

</div>
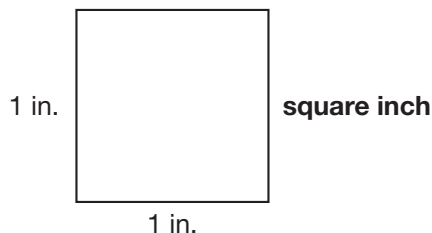
pulgada cuadrada	Medida de un área igual a la de un cuadrado con lados de 1 pulgada.

---

square root *(78)*	One of two equal factors of a number. The symbol for the principal, or positive, **square root** of a number is $\sqrt{}$.

*A **square root** of 49 is 7 because $7 \times 7 = 49$.*

$$\sqrt{49} = 7$$

raíz cuadrada	Uno de dos factores iguales de un número. El símbolo de la raíz cuadrada principal, o positiva, de un número es $\sqrt{}$. *La **raíz cuadrada** de 49 es 7, porque $7 \times 7 = 49$.*

---

statistics *(Inv. 5)*	A branch of mathematics that deals with the collection, analysis, organization, and display of numerical data.

*Some activities performed in **statistics** are taking surveys and organizing data.*

estadística	Una rama de las matemáticas que trata con la recolección, el análisis, la organización y la exhibición de los datos numéricos. *Algunas actividades que se llevan a cabo en **estadística** son hacer encuestas y organizar datos.*

| stem-and-leaf plot | A method of graphing a collection of numbers by placing the "stem" digits (or initial digits) in one column and the "leaf" digits (or remaining digits) out to the right. |

(Inv. 7)

```
Stem   Leaf
  2  | 1 3 5 6 6 8
  3  | 0 0 2 2 4 5 6 6 8 9
  4  | 0 0 1 1 1 2 3 3 5 7 7 8
  5  | 0 1 1 2 3 5 8
```

*In this **stem-and-leaf plot,** 3|2 represents 32.*

diagrama de hoja y tallos Un método para graficar una colección de números colocando los dígitos del "tallo" (o dígitos iniciales) en una columna y los dígitos de las "hojas" (o dígitos restantes) hacia la derecha.

*En este **diagrama de tallo y hojas,** 3|2 representa 32.*

straight angle
(31)

An angle that measures 180° and thus forms a straight line.
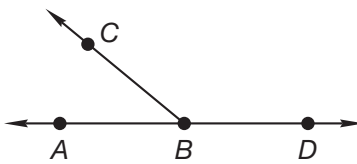
*Angle ABD is a **straight angle.** Angles ABC and CBD are not **straight angles.***

ángulo llano Un ángulo que mide 180° y por lo tanto forma una línea recta.

*El ángulo ABD es un **ángulo llano.** Los ángulos ABC y CBD no son **ángulos llanos.***

sum
(6)

The result of addition.

$7 + 6 = 13$ *The **sum** of 7 and 6 is 13.*

total de suma El resultado de la suma.

*La **suma** de 7 y 6 es 13.*

T

tally mark
(12)

A small mark used to help keep track of a count.

*I used **tally marks** to count cars. I counted seven cars.*

marca de conteo Una pequeña marca que se usa para llevar la cuenta.

*Usé **marcas de conteo** para contar carros. Yo conté siete carros.*

term
(1, 81)

1. A number in a sequence.

1, 3, 5, 7, 9, 11, …

*Each number in this sequence is a **term.***

2. A number that serves as a numerator or denominator of a fraction.

$\frac{5}{6}$ ⟩ **terms**

término **1.** Un número de una secuencia.

1, 3, 5, 7, 9, 11, …

*Cada número de esta secuencia es un **término.***

2. Número que se usa como numerador o denominador en una fracción.

tessellation *(Inv. 12)*	The repeated use of shapes to fill a flat surface without gaps or overlaps.
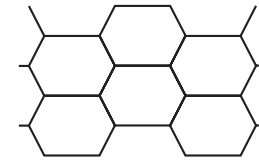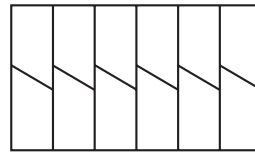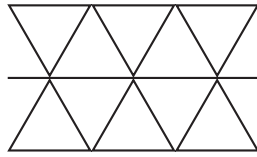
tessellations

mosaico	El uso repetido de figuras para llenar una superficie plana sin dejar huecos ni superposiciones.

tick mark *(12)*	Mark dividing a number line into smaller portions.
marca de un punto	Marca que divide una recta númerica en partes más pequeñas.

transformation *(Inv. 8)*	Changing a figure's position through rotation, reflection, or translation.

Transformations

Movement	Name
Flip	Reflection
Slide	Translation
Turn	Rotation

transformación	Cambio en la posición de una figura por medio de una rotación, reflexión o traslación.

translation *(Inv. 8)*	Sliding a figure from one position to another without turning or flipping the figure.

figure **translation** → image
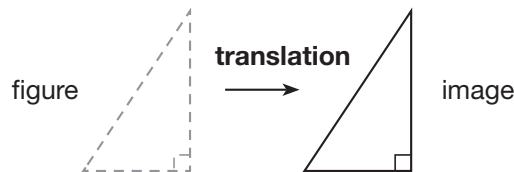
traslación	Deslizamiento de una figura de una posición a otra, sin rotar ni voltear la figura.

trapezium *(45)*	A quadrilateral with no parallel sides.

trapezium   not **trapeziums**

trapezoide	Cuadrilátero que no tiene lados paralelos.

trapezoid (45)	A quadrilateral with exactly one pair of parallel sides.

trapezoids   not **trapezoids**

trapecio	Cuadrilátero que tiene exactamente un par de lados paralelos.

---

triangle (32, 36)	A polygon with three sides and three angles.
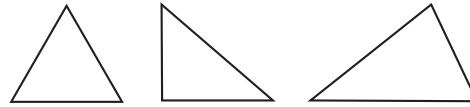
triangles

triángulo	Un polígono con tres lados y tres ángulos.

---

triangular numbers (50)	Numbers that can be represented by objects arranged in a triangular pattern.

3   6   10

Triangular numbers include all the numbers in this sequence:

$$+2 \quad +3 \quad +4 \quad +5 \quad +6$$
$$1, \quad 3, \quad 6, \quad 10, \quad 15, \quad 21, \ldots$$

números triangulares	Números que pueden ser representados por objetos arreglados en un patrón triangular. *Los **números triangulares** incluyen todos los números en esta secuencia.*

U

U.S. Customary System (44)	A system of measurement used almost exclusively in the United States. *Pounds, quarts, and feet are units in the **U.S. Customary System.***

Sistema usual de EE.UU.	Sistema de medición que se usa casi exclusivamente en EE.UU. *Libras, cuartos y pies son unidades del **Sistema usual de EE.UU.***

V

Venn diagram
(Inv. 7)

A diagram made of circles used to display data.
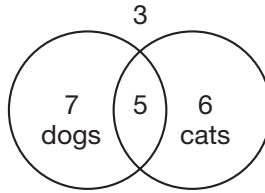
*This **Venn diagram** shows data on students' pets. Three students do not have a cat or a dog. Seven students have a dog, but not a cat. Six students have a cat, but not a dog. And five students have both a dog and a cat.*

diagrama de Venn

Un diagrama que utiliza círculos para mostrar datos.

*Este **diagrama de Venn** muestra los datos de las mascotas de estudiantes. Tres estudiantes no tienen gato ni perro. Siete estudiantes tienen perro pero no gato. Seis estudiantes tienen gato pero no perro. Y cinco estudiantes tienen tanto gato como perro.*

vertex
(32)

(Plural: *vertices*) A point of an angle, polygon, or solid where two or more lines, rays, or line segments meet.
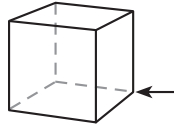
*The arrow is pointing to one **vertex** of this cube. A cube has eight **vertices.***

vértice

Punto de un ángulo, polígono o poliedro, donde se unen dos o más rectas, semirrectas o segmentos de recta.

*La flecha apunta hacia un **vértice** de este cubo está coloreado. Un cubo tiene ocho **vértices.***

vertical
(12)

Upright; perpendicular to horizontal.

vertical line

horizontal line   oblique line

not **vertical** lines
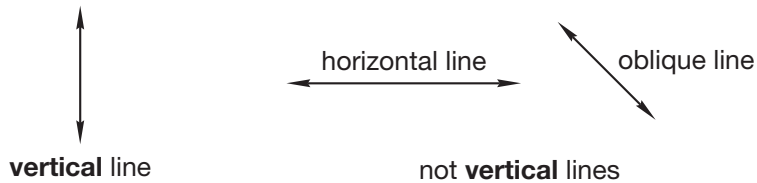
vertical

Perpendicular a la horizontal.

vertical axis
(Inv. 6)

The scale of a graph that runs from top to bottom.

eje vertical

La escala de una gráfica que corre de arriba hacia abajo.

volume (103)	The amount of space a geometric solid occupies. Volume is measured in cubic units.
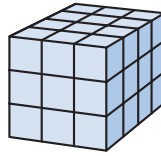
*This rectangular prism is 3 units wide, 3 units high, and 4 units deep. Its **volume** is 3 · 3 · 4 = 36 cubic units.*

volumen	Cantidad de espacio ocupado por un sólido geométrico. El volumen se mide en unidades cúbicas.

*Este prisma rectangular tiene 3 unidades de ancho, 3 unidades de altura y 4 unidades de profundidad. Su **volumen** es 3 · 3 · 4 = 36 unidades cúbicas.*

W

weight (77)	The measure of the force of gravity on an object. Units of weight in the customary system include ounces, pounds, and tons.

*The **weight** of a bowling ball would be less on the moon than on Earth because the force of gravity is weaker on the moon.*

peso	La medida de la fuerza de gravedad sobre un objeto. Las unidades de peso en el sistema usual incluyen onzas, libras y toneladas.

*El **peso** de una bola de boliche es menor en la Luna que en la Tierra porque la fuerza de gravedad es menor en la Luna.*

whole number(s) (2)	All the numbers in this sequence: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, ...

*The number 35 is a **whole number,** but $35\frac{1}{2}$ and 4.2 are not. **Whole numbers** are the counting numbers and zero.*

números enteros	Todos los números en esta secuencia: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 ....

*El número 35 es un **número entero** pero $35\frac{1}{2}$ y 4.2 no lo son.*
*Los **números enteros** son los números de conteo y el cero.*

X

***x*-axis** (Inv. 8)	The horizontal number line of a coordinate plane.
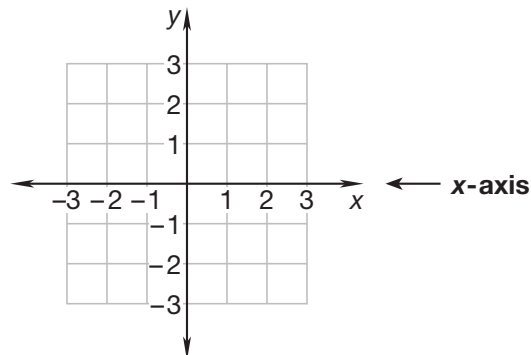
eje de las *x*	Recta numérica horizontal en un plano coordenado.

Y

y-axis
(Inv. 8)

The vertical number line of a coordinate plane.
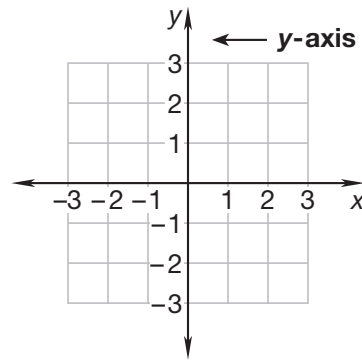
eje de las y   La recta numérica vertical en un plano coordenado.

Symbols

Symbol	Meaning	Example
$\triangle$	Triangle	$\triangle ABC$
$\angle$	Angle	$\angle ABC$
$\longrightarrow$	Ray	$\overrightarrow{AB}$
$\longleftrightarrow$	Line	$\overleftrightarrow{AB}$
$-$	Line segment	$\overline{AB}$
$\perp$	Perpendicular to	$AB \perp BC$
$\parallel$	Parallel to	$AB \parallel BC$
$<$	Less than	$2 < 3$
$>$	Greater than	$3 > 2$
$=$	Equal to	$2 = 2$
°F	Degrees Fahrenheit	100°F
°C	Degrees Celsius	32°C
$\llcorner$	Right angle (90° angle)	
…	And so on	1, 2, 3, …
$\times$	Multiply	9×3
$\cdot$	Multiply	$3 \cdot 3 = 9$
$\div$	Divide	$9 \div 3$
$+$	Add	$9 + 3$
$-$	Subtract	$9 - 3$
$\overline{)}$	Divided into	$3\overline{)9}$
R or r	Remainder	3 R 2
%	Percent	50%
x^2	"x" squared (times itself)	$3^2 = 3 \times 3 = 9$
x^3	"x" cubed	$3^3 = 3 \times 3 \times 3 = 27$
$\sqrt{\ }$	Square root	$\sqrt{9} = 3$ because $3 \times 3 = 9$.

Abbreviations

Abbreviation	Meaning
ft	Foot
in.	Inch
yd	Yard
mi	Mile
m	Meter
cm	Centimeter
mm	Millimeter
km	Kilometer
L	Liter
ml or mL	Milliliter
lb	Pound
oz	Ounce
kg	Kilogram
g	Gram
mg	Milligram
pt	Pint
qt	Quart
c	Cup
gal	Gallon

Formulas

Purpose	Formula
Perimeter of rectangle	$P = 2l + 2w$
Area of a square	$A = 4s$
Area of a rectangle	$A = l \cdot w$
Volume of a cube	$V = s^3$
Volume of a rectangular solid	$V = l \cdot w \cdot h$

Símbolos

Símbolo	Significado	Ejemplo
△	Triángulo	$\triangle ABC$
∠	Ángulo	$\angle ABC$
⟶	Rayo	$\overrightarrow{AB}$
⟷	Línea	$\overleftrightarrow{AB}$
─	Segmento de recta	$\overline{AB}$
⊥	Perpendicular a	$AB \perp BC$
‖	Paralelo a	$AB \parallel BC$
<	Menor que	$2 < 3$
>	Mayor que	$3 > 2$
=	Igual a	$2 = 2$
°F	Grados Fahrenheit	100°F
°C	Grados Celsius	32°C
∟	Ángulo recto (90° ángulo)	
…	Y así...	1, 2, 3, …
×	Multiplica	9×3
·	Multiplica	$3 \cdot 3 = 9$
÷	Divide	$9 \div 3$
+	Suma	$9 + 3$
−	Resta	$9 - 3$
⟌	Dividido entre	$3\overline{)9}$
R	Residuo	3 R 2
%	Porcentaje	50%
x^2	"x" cuadrada (por sí mismo)	$3^2 = 3 \times 3 = 9$
x^3	"x" cúbica	$3^3 = 3 \times 3 \times 3 = 27$
$\sqrt{}$	Raíz cuadrada	$\sqrt{9} = 3$ porque $3 \times 3 = 9$.

Abreviaturas

Abreviatura	Significado
pie	pie
pulg	pulgada
yd	yarda
mi	milla
m	metro
cm	centímetro
mm	milímetro
km	kilómetro
L	litro
ml or mL	mililitro
lb	libra
oz	onza
kg	kilogramo
g	gramo
mg	miligramo
pt	pinta
ct	cuarto
tz	taza
gal	galón

Fórmulas

Propósito	Fórmula
Perímetro de un rectángulo	$P = 2l + 2w$
Área de un cuadrado	$A = 4l$
Área de un rectángulo	$A = l \cdot w$
Volumen de un cubo	$V = l^3$
Volumen de un sólido rectangular	$V = l \cdot w \cdot h$

Hexagons, 199–200
  reflective symmetry of, 689

Histograms, 452. *See also* **Bar graphs**

Horizontal axis, 383, 450, 522

Horizontal bar graphs, 453. *See also* **Bar graphs**

Horizontal lines, 74

Hours, 171–173

Hundred thousands (place value), 40

Hundreds (place value), 18–19
  rounding to, 206

Hundredths (place value), 432–433
  decimal numbers, writing as, 425–427

I _____

I. *See* **Roman numerals**

Icons, 453

Identity Property
  of Addition, 34
  of Multiplication, 94, 511

Impossibility and chance, 360. *See also* **Probability**

Improper fractions, 486–487, 598–599
  multiplying, 784
  simplifying, 598–599
  whole numbers, changing to, 486–487
  writing mixed numbers as,
      743–744

Inch (in), 277–278, 479
  problems with feet, 295, 480
  square inch, 466

Integers, 74–76. *See also* **Cardinal numbers**;
  Counting numbers

International System of Units, 276. *See also* specific
  units of measurement

Intersecting lines and segments, 192–194

Inverse operations, 47, 120

Inverting fractions. *See* **Reciprocals**

Isosceles triangles, 223–224

Itineraries, 711–712. *See also* **Schedules**

K _____

Kilogram (kg), 499

Kilometer (km), 479

L _____

l. *See* **Length**

L. *See* **Roman numerals**

L (liter), 553–554

"Larger − smaller = difference" problems, 217–218

"Later − earlier = difference" problems, 218–219

lb. *See* **Pound**

LCM (least common multiple), 737

Leap years, 172

Least common multiple (LCM), 737

Legends (pictographs), 453

Length, 275–278
  of line segments, 390
  of rectangles, 340
  units of, 479–481

Less than (<), 23–24

Letters
  to identify angles and other geometric objects,
      388–390
  to represent numbers, 55–57, 88–89, 99, 114,
      134–135, 505, 748–751

Line graphs, 323, 383–384, 612

Line plots, 319–320

Line segments, 73–74, 192–194, 388. *See also* **Lines**
  length, 390
  oblique, parallel, and perpendicular, 193, 283
  in polygons, 199, 283

Lines, 73–74. *See also* **Line segments**; **Lines of
  symmetry**; **Number lines**; **Rays**
  intersecting, 192–194
  letters to identify lines, 388–390
  oblique, 74, 193
  pairs, 192–194
  parallel and perpendicular, 193–194

Lines of symmetry, 688–691

Liter (L), 553–554

Long division, 159–162, 263, 605–606. *See also*
  Division

Lowest terms of fractions, 527. *See also* **Fractions:
  reducing**

M _____

M. *See* **Roman numerals**

m (meter), 276, 413, 479

Mass, 498–500

Mean, 547, 549. *See also* **Average**

Measures of central tendency, 549. *See also* **Mean**;
  Median; **Mode**

Median, 320, 547–548, 549

Meter (m), 276, 413, 479

Metric system, 276. *See also* specific units of
  measurement
  as a base-ten system, 413, 480
  capacity, units of, 553–555
  decimal numbers and, 413–414
  length, units of, 479–481
  place value and, 413–414
  weight, units of, 499

Metric ton (t), 499

mg (milligram), 499

mi (mile), 277, 479–480

Middle (median), 320, 547–548, 549

Midnight, 173

INDEX

Triangular numbers, 311

Triangular prisms, 581

Turns, 524, 573–576. *See also* **Rotational symmetry**

Two-digit numbers
    dividing, 345–347, 605–606, 616–617
    multiplying, 326–329

U

Unit segments, 74–75

Units (ones), 333–335. *See also* **Ones**

U.S. Customary System, 277
    capacity, measures of, 553–555
    length, units of, 479–481
    weight, units, 499

V

V. *See* **Roman numerals**

Variables. *See* **Letters**

Venn diagrams, 454–455

Vertical axis, 383

Vertical lines, 74

Vertices, 199, 540

Volume, 671–674
    formula, 749–751

W

w. *See* **Width**

Weight, 498–500

Whole numbers, 11–13. *See also* **Counting numbers**
    adding, 33–36
        with decimal numbers, 644–646
    comparing, 23–24, 39–41
    decimal points and, 83
    dividing decimal numbers, 768–769
    divisibility of, 265–266
    factors, 154–156
    fractions
        multiplying by, 492–494, 559–561
        writing as, 401–402
    as halves of even numbers, 13
    improper fractions, changing from, 486–487
    multiplying by fractions, 492–494, 559–561
    naming, 28–30, 39–41
    ordering, 23
    reciprocals, 622–623
    rounding decimal numbers to nearest, 679–682
    subtracting fractions from, 401–402

Width, of rectangles, 340

Word problems. *See* **Problems**

Writing, and problem solving, 6

Writing numbers, 28–30, 39–41, 333–335, 432–433

X

X. *See* **Roman numerals**

x-axis, 522. *See also* **Horizontal axis**

Y

Yard (yd), 277, 479–480

y-axis, 522. *See also* **Vertical axis**

Years, 172

Z

Zero. *See* **0**

SAXON MATH™

There is a structure behind every high achiever.

Saxon Math is structured to help every student be a successful mathematics learner. It provides the time students need to discover, master, and apply mathematical concepts. The structure of *Saxon Math* puts students heads above the rest.

SAXON MATH™

SAXON™
A Harcourt Achieve Imprint

Hake

Harcourt Achieve